

Single-Issue Campaigns and Multidimensional Politics*

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Abstract

In most elections, voters care about several issues, but candidates may have to choose only a few on which to build their campaign. The information that voters will get about the politician depends on this choice, and it is therefore a strategic one. In this paper, I study a model of elections where voters care about the candidates' competences (or positions) over two issues, e.g., the economy and foreign policy, but each candidate may only credibly signal his competence or announce his position on at most one issue. Voters are assumed to get (weakly) better information if the candidates campaign on the same issue rather than on different ones. I show that the first mover will, in equilibrium, set the agenda for both himself and the opponent if campaigning on a different issue is uninformative, but otherwise the other candidate may actually be more likely to choose the other issue. The social (voters') welfare is a non-monotone function of the informativeness of different-issue campaigns, but in any case the voters are better off if candidates are free to pick an issue rather than if an issue is set by exogenous events or by voters. If the first mover is able to reconsider his choice when the follower picks a different issue, then politicians who are very competent on both issues will switch. If voters have superior information on a politician's credentials on one of the issues, that politician is more likely to campaign on another issue. If voters care about one issue more than the other, the politicians are more likely to campaign on the more important issue. If politicians are able to advertise on both issues, at a cost, then the most competent and well-rounded ones will do so. This possibility makes voters better informed and better off, but has an ambiguous effect on politicians' utility. The model and the results may help understand endogenous selection of issues in political campaigns and the dynamics of these decisions.

Keywords: Elections, campaigns, issues, dynamics, salience, competence, probabilistic voting, unraveling.

JEL Classification: D72, D82.

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Campaigns are not like school debates or courtroom disputes. <...>
Campaigns are quite different because there is no judge. Each disputant decides what is relevant, what ought to be responded to, and what themes to emphasize.

William H. Riker¹

1 Introduction

In most elections, voters care about multiple issues and, similarly, parties and individual candidates write platforms stating their positions on many issues and spanning dozens of pages. Yet, on the campaign trail, in political advertisements, and in debates, it is typical for candidates to focus on a narrow subset of issues and reiterate the same points over and over. This strategy makes a lot of sense if voters pay relatively little attention to the election—for example, if a typical voter will only see a few ads or attend one rally. The candidates are free to choose their campaign themes, but the quality and informativeness of the debate may depend on the issues they choose.² In this paper, I study informative campaigns with endogenous choice of issues by the candidates.

I assume that voters' preferences are two-dimensional (and separable), and there are two politicians who choose the issues they will run on. Two assumptions are key. First, I assume that a politician can only credibly announce his position on (at most) one issue, but not on both. The motivation is that persuading voters about his position or about his talents in a particular area is hard, and there is only a limited amount of time. If voters' attention is limited, losing focus is costly for the candidate. In reality, even though candidates occasionally talk about several issues, the broad idea of the campaign is typically clear: for example, Bill Clinton ran on the economy in 1992, while George H.W. Bush ran on foreign policy; in 1972, George McGovern ran against the Vietnam war. Sometimes candidates shift their focus during the campaign, as in 2008, when the financial crisis made the economy—as opposed to foreign policy and, specifically, the Iraq war—the most salient issue, which subsequently drew the attention of both Barack Obama and John McCain, or in 2012, when the Romney campaign primarily attacked Obama's economic record, and Obama eventually moved from focusing on social issues to defending the economic record of his first term. Still, the assumption that at each stage (e.g., in a given rally or debate) the candidate adopts a particular line of attack or defense, which may or may not coincide with the one chosen by his opponent, seems reasonable.³

¹William H. Riker (1993), "Rhetorical Interaction in the Ratification Campaigns," in *Agenda Formation*, ed. by William H. Riker. The University of Michigan Press: Ann Arbor, p.83.

²In a book on candidates' behavior in American politics, Simon (2002) emphasizes the informational value of dialogue in political campaigns, but notes that dialogues are rarely observed.

³Campaign advisers have long known the importance of focusing on very few issues and very few talking points. Matalin and Carville (1994) describe the advice they gave Clinton in the 1992 campaign: "Governor Clinton didn't

The second key assumption I make is that voters get better information about a candidate’s position or competence on a given issue if both candidates choose this issue. In other words, there is a certain complementarity: a politician sounds more credible if he talks about, say, the economy, if his opponent talks about the same issue as well. Indeed, the politicians may criticize each other, check whether the opponent’s factual statements are correct, and thus indirectly add credibility to each other’s claims. On the other hand, if the opponent talks about another topic, then the politician’s own credibility is undermined. The process of making statements and having the other party check their veracity is not modeled in the paper explicitly, but the assumption that voters learn more if politicians have a sensible debate on the same issue seems natural.⁴

These two assumptions lead to a simple and tractable model of issue selection. I first study a basic case where the first mover (termed “Incumbent” for clarity and brevity) commits to building his campaign on one of the two issues. The other politician (“Challenger”) observes this and decides whether to reciprocate or talk about the other issue instead. Voters are Bayesian, they update on the politicians’ decisions and the information that they observe, and vote, probabilistically, for one of the two candidates. The model gives the following predictions. First, if campaigning on different issues gives politicians very little credibility, there is a strong unraveling effect, which forces Challenger to respond to Incumbent and talk about the same issue. However, if a politician is quite likely to be able to credibly announce his position or establish his competence even when talking about a different issue, then divergence is possible, and it is in fact more likely that Challenger will choose a different issue. Second, social welfare (as measured by the expected competence of the elected politician) does not necessarily increase in the ability of politicians to make credible announcements when the opponent campaigns on a different issue. The reason is that when this ability is low, the politicians will choose the same issue in equilibrium, and this will help voters rather than hurt them. Third, despite loss of information if candidates campaign on different

want to cede any issues to Perot. He said, ‘I’ve been talking about these things [the role of government] for two years, why should I stop talking about them now because Perot is in?’ Our response, and it was not easy to confront the governor with this, was, ‘There has to be message triage. If you say three things, you don’t say anything. You’ve got to decide what’s important.’”

⁴One way to microfound this assumption is by introducing a third-party fact-checker who is active only some of the time; however, if both candidates campaign on the same issue, their opponent fills in this role automatically. This microfoundation would be realistic: for example, in 2008, Obama rebutted McCain’s attempt to separate himself from the incumbent, George W. Bush, by constantly noting that McCain had voted with Bush 90 percent of the time. An alternative way is to assume that political positions of politicians are correlated, and when one candidate states his position, voters update on the other candidate’s position as well. For example, Barack Obama’s open support of gay marriage might be interpreted as a belief shared by all politicians of his generation and thus have little impact on voters’ decision to support or oppose him—unless his opponent takes a clear stance, too. In this case, again, voters will receive more precise signals about the *difference* in the candidates’ positions when they campaign on the same issue. Another alternative is to assume that each candidate has a large set of favorable and unfavorable facts and arguments, and if only one politician talks about an issue, the voters only see the facts that favor his position, while if both campaign on an issue, the voters get the full picture (see, for example, Dziuda, 2011, on selective use of arguments by a biased expert). I do not model either mechanism explicitly in order to focus on the consequences of endogenous issue selection.

issues, fixing an issue for both candidates to campaign on will generally hurt voters. Fourth, if the first mover (Incumbent) is allowed to reconsider his initial choice of issue (e.g., because the opponent picked a different issue, or because the campaign is long enough), then switching focus is not a signal of weakness; rather, it signals competence in both issues. Fifth, if voters are more informed on Incumbent's competence in one of the issues, say, the economy, then he is more likely to campaign on the other issue, on which the voters have less information with respect to the Incumbent's level of competence. In practice, this implies that Incumbent is likely to run for reelection on an issue different than the one with which he had a chance to demonstrate his competence (or incompetence) during the first term; doing the opposite would be interpreted as a lack of competence in the other area. Finally, if politicians have some discretion whether to be the first or second to start a campaign, they will likely postpone a campaign, as this would signal their relative indifference between the two issues. To the best of my knowledge, this paper is the first to model the dynamics of issue selection in political campaigns.

Apart from the applied results described above, the model has a theoretical appeal. On the one hand, despite being parsimonious, with only two individuals making binary decisions, the model is rich enough to give rise to this broad set of implications which are potentially testable. On the other hand, for a signaling model with four dimensions of uncertainty (two for each candidate), the model is surprisingly tractable. It also suggests a novel limitation for the standard unraveling argument (Grossman and Hart, 1980, Grossman, 1981, Milgrom, 1981), which would be applied to this model of political campaigns in the following way: if one candidate chose to talk about, say, the economy, then avoiding this issue would signal the lowest possible competence on this dimension, and therefore all types, except perhaps the very worst ones, would choose the same dimension. As this paper shows, this argument may fail if there is an alternative statement that a person can make; in this case, the benefits of making such a statement may outweigh the cost of giving the wrong perception, and this might destroy the unraveling equilibrium. More generally, this logic is one of opportunity cost: if disclosing some private information involves an opportunity cost (e.g., of time), then the higher this cost (the more valuable the opportunity is), the less unraveling one can expect.

This reasoning is applicable to one particular aspects of political campaigns: debates. It can help explain why and when candidates dodge questions in the course of a debate, and why they often get away with that. This paper suggests the following: if a candidate dodged the question and proceeded with saying something unimportant, he will be punished severely by the voters. At the same time, if he made an important statement instead, the voters' opinion about the candidate's ability or position regarding the first question will become worse, but only mildly so, and overall, seizing the opportunity to make an important statement may benefit the candidate. Interestingly, the better the candidate performs with respect to the other statement he chooses to make, the less

the voters will punish him with respect to the original question, as they will believe that dodging the question was justified.⁵

The strategic choice of campaign issues by politicians has been the topic of a number of descriptive and formal studies. Riker (1996, see also 1993), in an extensive study of the U.S. Constitution ratification, observes that politicians are likely to abandon an issue in which they cannot beat their opponent; this means that debating the same issue should rarely be observed. This observation predicts “issue ownership” (as in Petrocik, 1996; see also Petrocik et al., 2003; Simon, 2002, contains a model that captures this idea). Aragonès, Castanheira, and Giani (2015) study a model where parties compete by investing in generating high-quality alternatives to the status quo. Their model can explain issue ownership whereby parties invest in issues on which they have comparative advantage on; however, if voters are sufficiently susceptible to priming, “issue stealing” is also possible. Other papers that model political campaigns as advertisements that raise the salience of an issue include Amorós and Socorro Puy (2007), which predicts issue convergence if one party has an absolute advantage on two issues but little comparative advantage, and Colomer and Llavador (2011), where a challenger proposes an alternative policy on one issue and the incumbent then has a choice on whether to defend the status quo or campaign on a different issue; the latter paper predicts issue convergence if voters like the status quo enough, but otherwise divergence is possible. In Dragu and Fan (2013), politicians need to divide a fixed budget among several issues; the authors show that more popular parties are likely to campaign and thereby increase the salience of consensual issues, while less popular parties increase the salience of divisive issues. In Ash, Morelli, and Van Weelden, campaigning on a divisive issue (“posturing”) signals commitment to a policy position on that issue but can reduce social welfare because of the neglect of important consensual issues. Berliant and Konishi (2005) consider candidates who are able to take any positions on any subset of policy dimensions; they argue that both candidates at least weakly prefer to campaign on all issues, but this result ceases to hold in the presence of Knightian uncertainty.

The paper most closely related to this one is Polborn and Yi (2006). There, the voters are also Bayesian and have fixed preferences. The politicians also choose to disclose information on one of two dimensions: positive information about themselves or negative information about the opponent. The authors characterize a unique equilibrium, in which running a negative campaign reveals a lack of positive information about oneself. This paper generalizes Polborn and Yi (2006) for the case

⁵The insights of the paper make it potentially applicable to advertising campaigns. For example, if Apple released a new iPhone and marketed it with an emphasis on the screen resolution, then Samsung would face a strategic choice. It could advertise the screen resolution of its new Galaxy phone, which would allow potential users to make a direct comparison, or it could advertise the phone’s battery life. The intuition from this paper suggests that either decision may be optimal depending on the relative strength of the new device along these two dimensions, and also that the optimization problems of the first mover (Apple in this example) and the second mover (Samsung) are different. Of course, in the marketing application, another important choice variable is price; however, one can expect the main intuition to hold. The results may even be directly applicable if the price has to be fixed at some round number, such as \$599 or \$699.

of generic campaign issues where campaigning on different issues may undermine voters' ability to learn the truth.⁶ As it turns out, this generalization removes the complete separability of the two politicians' problems while preserving tractability; this makes it possible to obtain nontrivial and realistic predictions about the choice of issues and campaign dynamics.⁷

The model in this paper assumes that the relevant characteristic of candidates is their competence in each of the two issues, and all voters have the same preferences (greater competence is better), but similar forces would be in effect if candidates were competing on more divisive issues; in the latter case, the counterpart of competence would be the proximity of a candidate's ideal point (in a given policy dimension) to the median voter's position. The model would predict that politicians would have an incentive to campaign on an issue where their position is close to that of the median voter, and if a politician revealed himself to be distant from the median voter on the issue he chose, voters would suspect that he is even more radical on the other issue. The results would be valid under the following assumptions: the candidates cannot commit to any policy position other than their ideal one in the course of the campaign, and they cannot lie about their position (or, more precisely, they cannot lie if the other party is campaigning on the same issue and is able to expose the lie to voters).⁸ In the current model, voters' preferences are aligned and competence is unambiguously good, so pandering must take the form of exaggerating one's competence. The results are driven by the assumption that doing so is easier if the opponent talks about a different issue; the assumption that exaggeration is either infinitely costly (or at least that competence is

⁶Mattes (2007) also considers the possibility of negative campaigns and argues that the welfare effect of banning them would be ambiguous. Several papers study strategic revelation of hard evidence in debates. In Chen and Olszewski (2014), discussants have a choice between strong and weak arguments, and the authors show that committing to a weak argument may sometimes be desirable. In an earlier paper, Austen-Smith (1990) models debates in legislatures as cheap talk and argues that they affect outcomes only through affecting agenda-setting, but not voting.

⁷There are several related studies of informative campaigns where voters are Bayesian, but candidates do not face the strategic choice of which issue to campaign on. In Prat (2002), voters observe advertisement spending and view this information as evidence of endorsement by an interest group that provided the campaign finance. In Coate (2004a,b), advertisements transmit information about the candidates' positions on a policy issue (as in this paper, the candidates cannot lie). The amount of advertisements, and thus the number of citizens they cover, is chosen by the candidate and may depend on his political position. Thus, voters who failed to see an advertisement by a candidate infer his political position in a Bayesian way. See also Galeotti and Mattozzi (2011) for a model where voters who saw an advertisement may transmit this information along a network.

⁸There is a large literature on pandering to voters by partisan politicians as well as obscuring one's positions, both on the campaign trail and in office, starting with Shepsle (1972), who argues that in the presence of voters with local risk aversion, equilibria with imperfect revelation of political positions are possible. Alesina and Cukierman (1990) suggest that incumbents have an incentive to be ambiguous (see also Heidhues and Lagerlof, 2003). Glazer and Lohmann assume that maintaining flexibility on an issue, as opposed to committing to a policy, leaves the issue salient and may therefore benefit the politician. Callander and Wilkie (2007) talk about lying on the campaign trail, as does Bhattacharya (2011). Kartik and McAfee (2007) consider signaling motive in policy choices; Acemoglu, Egorov, and Sonin (2013) suggest that signaling may make politicians choose policies further from the median voter's ideal point rather than closer to it, thus explaining the phenomenon of populism. Relatedly, Kartik and Van Weelden (2014) show that during campaigns, a politician may reveal himself to be noncongruent as a way to commit to not pandering in the future, or may decide not to do so and leave open the possibility that he is congruent; the campaigns in their paper feature informative cheap talk about politicians' types that nevertheless leaves voters indifferent as to whom to elect.

fully revealed) or totally uncontrolled to the point that the candidate has zero credibility makes the model tractable, but hardly drives the results.⁹

The rest of the paper is organized as follows. Section 2 introduces the basic model. Section 3 studies the equilibria of the basic model, with sequential or simultaneous moves, and obtains implications for social welfare and election outcomes. In Section 4, a dynamic version is introduced, where each politician has a chance to respond to the other’s choice of issue. Section 5 discusses several extensions of the basic model. Section 6 concludes. Appendix A contains the main proofs; Appendix B (not for publication, to be made available online) contains auxiliary proofs and treats some out-of-equilibrium cases.

2 Model

Consider a two-dimensional policy space, one dimension being economy (E) and the other being foreign policy (F). There are two politicians, referred to as Incumbent, indexed by i , and Challenger, indexed by c ; this is mainly done for brevity, and otherwise the politicians will be symmetric until this assumption is relaxed in Section 5.1. Each politician $j \in \{i, c\}$ has a two-dimensional type $a_j = (e_j, f_j)$, which corresponds to his ability in economic and foreign policy questions, respectively (in what follows, $a|_s$ will denote the projection of a on issue $s \in \{E, F\}$). Consider an electorate with perfectly aligned preferences: there is a continuum of voters, and the utility of each voter if politician of type (e, f) is elected is

$$U(e, f) = e + f; \tag{1}$$

I thus assume that voters weigh both issues equally (this assumption is relaxed in Section 5.2). The type of each politician j is his private information and is not known to the other politician or voters; the distribution of types, independent and uniform on $\Omega_j = [0, 1] \times [0, 1]$, is common knowledge. At the time of voting, all voters have the same information on both Incumbent and Challenger: they know the history of the candidates’ moves as well as the moves of Nature. This information (or, more precisely, the posterior distribution of (e_i, f_i, e_c, f_c) conditional on the history) is denoted by \mathcal{I} for brevity. Voting is probabilistic (see, e.g., Persson and Tabellini, 2000): voter k votes for Incumbent if and only if

$$\mathbb{E}(U(e_i, f_i) - U(e_c, f_c) \mid \mathcal{I}) > \theta + \theta_k, \tag{2}$$

⁹The paper also addresses the question on whether a single political dimension is likely to arise endogenously in a multidimensional world. If it is, this would be further justification for studying political competition along a single dimension, in the same way as Duverger’s law (Riker, 1982; see also Lizzeri and Persico, 2005) justifies studying political competition between two candidates by predicting the emergence of exactly two candidates in a majoritarian system. This question is also studied in McMurray (2013); see also Duggan and Martinelli (2011) who study a model of media slant, where media are assumed to collapse a multidimensional policy position to a one-dimensional one.

where θ is a common taste shock and θ_k is voter k 's individual shock. As is standard, I assume that θ is distributed uniformly on $[-\frac{1}{2A}, \frac{1}{2A}]$ and θ_k distributed uniformly on $[-\frac{1}{2B}, \frac{1}{2B}]$, where $A < \frac{1}{2}$ and $B < \frac{A}{2A+1}$.

During the campaign, each politician can only talk about one issue, economy or foreign policy. If both talk about the same issue, they have a reasonable conversation or debate, during which the voters perfectly learn their competences in this dimension (e_i and e_c , or f_i and f_c). However, if the politicians end up campaigning on different issues, the voters find out the politicians' competences with probability μ only. In other words, a low μ means that it is hard for a politician to make credible statements without being engaged in a sensible debate with the opponent or, alternatively, one can think of this as noisy communication between politician and voters when there is nobody around to limit exaggeration or bluffing. In particular, $\mu = 0$ corresponds to zero credibility, and $\mu = 1$ is the other extreme where a politician's ability to make credible statements about his competence does not depend on his opponent's choice of campaign issue.¹⁰ For simplicity, assume $\mu > 0$; the extreme case $\mu = 0$ is relatively uninteresting, but would require special treatment in many of the proofs.

More precisely, the types of Incumbent and Challenger are given, respectively, by $a_i = (e_i, f_i) \in \Omega_i$ and $a_c = (e_c, f_c) \in \Omega_c$; they are assumed to be independent and distributed uniformly on their respective domains, which we for now assume (for simplicity) to be identical: $\Omega_i = \Omega_c = [0, 1]^2$. Incumbent moves first and chooses the issue to campaign on, $d_i \in \{E, F\}$; Challenger observes this and chooses his issue $d_c \in \{E, F\}$. In other words, the set of Incumbent's strategies is $S_i : \Omega_i \rightarrow \{E, F\}$ and the set of Challenger's strategies is $S_c : \Omega_c \times \{E, F\} \rightarrow \{E, F\}$.¹¹ Nature then decides whether each of the candidates is successful in announcing his competence, and voters get signals $\kappa_i, \kappa_c \in [0, 1] \cup \{\emptyset\}$ about the politicians' competence on the issues they cover in their campaigns. Thus, if $d_i = d_c$, then $\kappa_i = a_i|_{d_i}$ and $\kappa_c = a_c|_{d_c}$; if, however, $d_i \neq d_c$, then with probability μ voters get the same signals $\kappa_i = a_i|_{d_i}$ and $\kappa_c = a_c|_{d_c}$, and with probability $1 - \mu$, $\kappa_i = \kappa_c = \emptyset$; let us write $\chi = 1$ in the first case and $\chi = 0$ in the second.¹² Each voter observes the history of moves (d_i, d_c) and signals (κ_i, κ_c) , and updates accordingly, thus getting conditional distribution \mathcal{I} , which allows

¹⁰This signal structure may be microfounded in the following way. Assume that a politician may announce any competence, but is heavily penalized if he is found to have exaggerated. When politicians talk about the same issue, there is only chance μ that some third party is willing to check the politician's announcements. However, when they talk about the same issue, there is always someone to do this, and in this case the politicians have to be credible, so voters learn the true competences on this issue. On the other hand, if nobody puts a check on politicians, then all announce that they are the most competent, and Bayesian voters learn nothing. I do not model this explicitly to simplify the exposition.

¹¹It should be emphasized that the two candidates do not have private information about the types of their opponent, and, in particular, Challenger makes his decision knowing the issue that Incumbent chose, but not Incumbent's competence in that issue. To put it another way, I assume that politicians have as much information about their opponents as the voters. This is a simplification of reality, but a helpful one: from a technical standpoint, it prevents politicians from strategically jamming the opponent's signal if they know it to be very high.

¹²Alternatively, one could assume that whether $\kappa_i = \emptyset$ and $\kappa_c = \emptyset$ is determined independently; this does not affect the analysis.

him to compute the difference in competence between Incumbent and Challenger:

$$D(d_i, d_c, \kappa_i, \kappa_c) = \mathbb{E}(U(e_i, f_i) - U(e_c, f_c) \mid d_i, d_c, \kappa_i, \kappa_c) = \mathbb{E}(U(e_i, f_i) - U(e_c, f_c) \mid \mathcal{I}). \quad (3)$$

After that, common shock θ and idiosyncratic shocks θ_k are realized for each voter k , and voter k votes for Incumbent if and only if (2) holds.

Both politicians are expected utility maximizers. The utility of each is normalized to 0 if he is not elected and to 1 if he is elected, and therefore each maximizes the probability of being elected. The equilibrium concept is the following refinement of the standard Perfect Bayesian equilibrium (PBE) in pure strategies: voters' beliefs are such that if politician $j \in \{i, c\}$ (Incumbent or Challenger) chooses E , then his expected utility is nondecreasing in his competence on economy, e_j , and if he chooses F , then it is nondecreasing in f_j .¹³ This simply means that neither politician would be willing to understate his competence if given this option. In Appendix B, Lemma B2 proves that monotone PBE must exhibit 'monotonicity in strategies': namely, if Incumbent's strategy satisfies $d_i(e_i, f_i) = E$ for some (e_i, f_i) , then $d_i(e'_i, f_i) = E$ for $e'_i > e_i$ and $d_i(e_i, f'_i) = E$ for $f'_i < f_i$, and similar requirements are satisfied for Challenger's decisions $d_c(e_c, f_c; d_i = E)$ and $d_c(e_c, f_c; d_i = F)$, which simplifies the analysis considerably. Finally, I do not distinguish between equilibria that differ on a subset of types of measure zero; for example, if all incumbents with $e_i = f_i$ are indifferent between economy and foreign policy and can choose either way, this is treated as a single equilibrium.

3 Analysis

In this section, we analyze the game introduced in Section 2. Then, we consider an alternative story where both candidates choose issues simultaneously. We conclude this Section by studying social welfare and compare the results for different values of the credibility parameter μ and for both timings.

Let us first compute the probability of each politician to be elected for any possible posterior distribution $\mathcal{I} = \mathcal{I}(d_i, d_c, \kappa_i, \kappa_c)$. For a given θ , citizen k votes for Incumbent with probability

$$\Pr(\theta_k < D(\mathcal{I}) - \theta) = \frac{1}{2} + B(D(d_i, d_c, \kappa_i, \kappa_c) - \theta), \quad (4)$$

where $D(\mathcal{I})$ is the difference of voters' expectations of the politicians' competences given by (3); this is also the share of votes Incumbent gets. Incumbent wins if and only if (4) exceeds $\frac{1}{2}$, which happens with probability

$$\Pr(\theta < D(\mathcal{I})) = \frac{1}{2} + AD(\mathcal{I}). \quad (5)$$

¹³Polborn and Yi (2006) introduce a similar monotonicity refinement in a game where politicians choose to run a positive campaign or a negative campaign. The focus on pure strategies is without loss of generality in the model, but is done to save on notation and to simplify the proofs.

Since A is a constant, Incumbent and Challenger seek to maximize and minimize the expectation of $D(\mathcal{I})$, respectively. They thus solve, respectively,

$$\max_{d_i} \mathbb{E}(D(\mathcal{I}) | e_i, f_i) = \mathbb{E}(\mathbb{E}(U(e_i, f_i) - U(e_c, f_c) | \mathcal{I}) | e_i, f_i) \text{ and} \quad (6)$$

$$\min_{d_c} \mathbb{E}(D(\mathcal{I}) | e_c, f_c; d_i) = \mathbb{E}(\mathbb{E}(U(e_c, f_c) - U(e_c, f_c) | \mathcal{I}) | e_c, f_c; d_i), \quad (7)$$

where the exterior expectations are politicians' at the time of their decision-making, and the interior ones are voters'.

Let us examine the politicians' problems, (6) and (7), more closely. Notice first that at the time either politician makes a decision, he knows that he can affect both $\mathbb{E}(U(e_i, f_i) | \mathcal{I})$ and $\mathbb{E}(U(e_c, f_c) | \mathcal{I})$, i.e., the voters' posteriors both about himself and about his opponent (e.g., Challenger can make a very competent Incumbent appear worse if he chooses a different issue, because Incumbent may then fail to inform the voters about his competence). However, if he takes the expectation of voters' posterior regarding his opponent *conditional only on the information he knows at the time of decision-making*, he will get the current expectation (both his and the voters') of the opponent's competence, which he cannot change. This greatly simplifies the problem by effectively separating the problems of Incumbent and Challenger:

Lemma 1 *In equilibrium, Incumbent and Challenger maximize, respectively,*

$$\max_{d_i} \mathbb{E}(\mathbb{E}(U(e_i, f_i) | d_i, \kappa_i) | e_i, f_i), \quad (8)$$

and

$$\max_{d_c} \mathbb{E}(\mathbb{E}(U(e_c, f_c) | d_c, \kappa_c; d_i) | e_c, f_c; d_i). \quad (9)$$

Proof. *The complete proof of this and other results are in Appendix A. ■*

The properties of equilibrium critically depend on whether μ exceeds $\frac{1}{2}$ or not: for $\mu > \frac{1}{2}$, unraveling does not happen in equilibrium, whereas for $\mu \leq \frac{1}{2}$ unraveling may happen. The next Proposition characterizes the equilibrium for a relatively high μ .

Proposition 1 *If $\mu > \frac{1}{2}$, there is a unique equilibrium. In this equilibrium, Incumbent chooses the issue that he is more competent in: economy if $e_i > f_i$ and foreign policy if $f_i > e_i$ (and he is indifferent otherwise). If $d_i = E$, then Challenger chooses E if and only if*

$$e_c > \frac{2\mu - 1}{2 - \mu} f_c + C, \text{ where } C = \frac{4 - 5\mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)}, \quad (10)$$

and if $d_i = F$; then a symmetric formula applies.

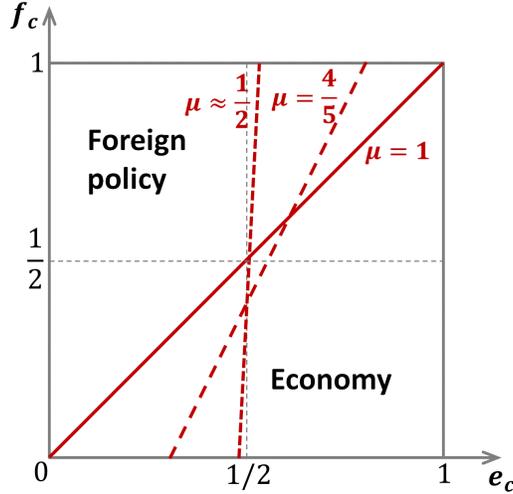


Figure 1: Challenger's equilibrium response if Incumbent picked Economy; $\frac{1}{2} < \mu \leq 1$.

Not surprisingly, Incumbent always chooses the issue that he is most competent in. The Challenger's response depends on the chance that he will be understood if he chooses a different issue. If $\mu = 1$, then his credibility is the same regardless of the issue, and then his strategy is independent from the choice of Incumbent: he will always choose the issue his is most competent in. If $\frac{1}{2} < \mu < 1$, then he may not be as credible when choosing a different issue. For a very competent (in both dimensions) Challenger, this gives a reason to choose the same issue as Incumbent, in order to signal his competence in either issue and avoid being pooled with the less competent mass of potential challengers. Conversely, if Challenger lacks competence in both dimensions, he is better off choosing an issue different from Incumbent's choice, because he will have a better chance to be viewed as an average rather than a very bad type. In equilibrium, if Incumbent chose Economy, then the set of Challengers who are indifferent between the two alternatives is a straight line that is steeper than the diagonal (see Figure 1). In other words, the choice of Challenger is more sensitive to e_c than to f_c , which is not surprising since he has a higher chance to communicate e_c than f_c credibly. For example, if μ is close to $\frac{1}{2}$, then Challenger's choice depends almost exclusively on e_c .

We therefore have the following equilibrium strategies of the politicians. Incumbent always chooses the issue he is best at. Challengers who are good at one issue and bad at the other also choose their preferred issue, regardless of the choice of Incumbent. However, Challengers which have roughly equal abilities in both dimensions and who would otherwise be relatively indifferent respond to Incumbent's pick of issue in a non-trivial way: those who excel in both dimensions pick the same issue, whereas very incompetent ones choose a different dimension. The equilibrium strategies are summarized in Figure 2. Notice that for μ close to 1, the lines separating the four regions converge to the diagonal, and Challenger's decision becomes largely independent from that of Incumbent. Conversely, if μ is close to $\frac{1}{2}$, the four regions have (almost) equal size, and half of

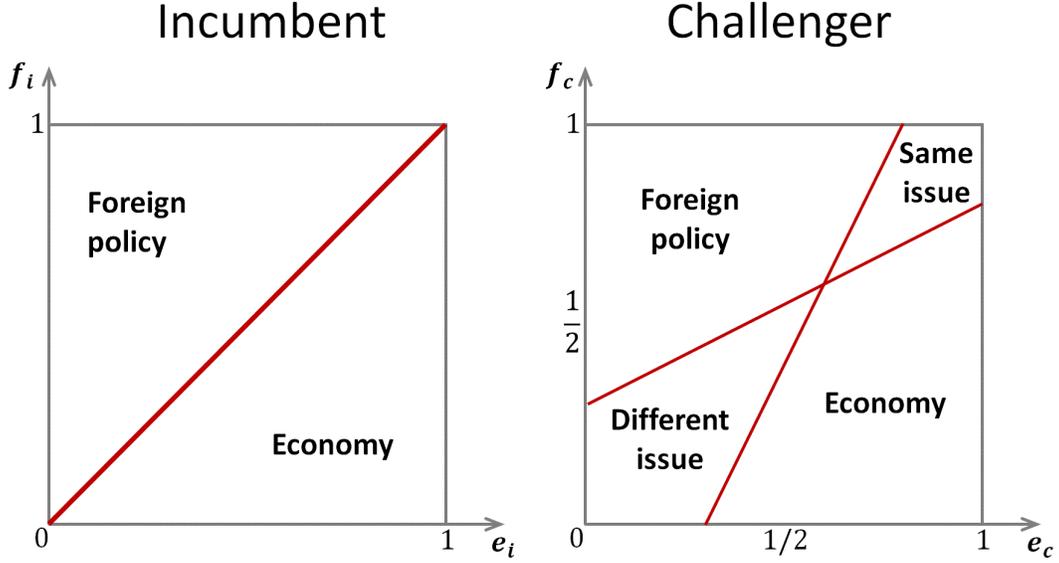


Figure 2: Equilibrium strategies of Incumbent and Challenger if $\frac{1}{2} < \mu < 1$.

Challengers will condition their choice of issue on that of Incumbent (the upper-right corner will always pick the same issue and the lower-left one will always pick the other issue).

The formal proof of Proposition 1 is in Appendix A, but the idea is relatively straightforward. The Incumbent's problem is symmetric, and thus it is natural to expect symmetric equilibrium strategies. Hence, let us focus on Challenger. For simplicity, suppose that both issues E and F are picked with a positive probability, and suppose that Challenger with type $(e_c, f_c) = (x, y)$ is indifferent, whereas those with a higher e_c or lower f_c choose E and those with lower e_c or higher f_c choose F (Appendix A fills in the details on why such equilibrium exists and why there are no other equilibria). Suppose that Incumbent chose E ; then, if this Challenger with type (x, y) chooses E , the voters will perceive him as having total competence $x + \hat{f}_c(x)$, where $\hat{f}_c(x) = \mathbb{E}(f_c \mid d_i = E, d_c(e_c, f_c) = E, e_c = x) = \frac{y}{2}$. If he chooses F , then with probability μ voters will perceive his total competence as $y + \hat{e}_c(y)$, where $\hat{e}_c(y) = \mathbb{E}(e_c \mid d_i = E, d_c(e_c, f_c) = F, f_c = y) = \frac{x}{2}$. With complementary probability $1 - \mu$, however, they will only know that he chose F , and will think of him as $\mathbb{E}(e_c + f_c \mid d_i = E, d_c(e_c, f_c) = F)$, which we denote by \tilde{a}_c^F . Then the indifference condition for Challenger of this type may be written as

$$x + \frac{y}{2} = \mu \left(y + \frac{x}{2} \right) + (1 - \mu) \tilde{a}_c^F, \quad (11)$$

and thus we get an upward-sloping boundary between E and F with a slope $\frac{2-\mu}{2\mu-1}$ (this is positive if $\mu > \frac{1}{2}$). Following the intuition above, conjecture that Challenger of type $(1, 1)$ chooses E and one of type $(0, 0)$ chooses F ; then (11) intersects the boundary of Ω_c at points $(\alpha, 0)$ and $(\beta, 1)$ for some α, β . If so, one can compute \tilde{a}_c^F directly: $\tilde{a}_c^F = \frac{\alpha^2 + \alpha\beta + \alpha + \beta^2 + 2\beta}{3(\alpha + \beta)}$. Substituting this into (11),

we have an equation on (x, y) which should hold for $(\alpha, 0)$ and $(\beta, 1)$. It is then straightforward to find that $\alpha = \frac{4-5\mu+\sqrt{\mu(8-7\mu)}}{4(2-\mu)}$, $\beta = \frac{3\mu+\sqrt{\mu(8-7\mu)}}{4(2-\mu)}$, which implies (10).

Are the two politicians ex ante more likely to campaign on the same issues or on different issues? Surprisingly, for $\mu \in (\frac{1}{2}, 1)$, the answer is that campaigning on different issues is more likely (and this is captured on Figure 2): Challenger is more likely to choose economy if Incumbent chose foreign policy, and vice versa. More precisely, we have the following result.

Proposition 2 *If $\mu > \frac{1}{2}$, then the probability of having politicians talk about different issues, is higher than $\frac{1}{2}$: it increases in μ on $(\frac{1}{2}, \frac{4}{5})$ and decreases on $(\frac{4}{5}, 1)$, thus reaching its maximum at $\mu = \frac{4}{5}$. Nevertheless, the probability that either politician successfully communicates his competence in the chosen dimension is strictly increasing in μ .*

It is true that politicians who choose the opposite issue have lower expected quality, and thus candidates have an incentive to pool with those who choose the same issue as Incumbent. To get the intuition for this result, however, consider politicians who are relatively equally competent (or incompetent) on both issues. Campaigns where one can reveal one's competence in one issue but not the other punish such politicians disproportionately harshly, and these types are key to determine whether Challenger is more likely to choose the same or a different issue. In particular, consider Challenger of type $(\frac{1}{2}, \frac{1}{2})$ and suppose for a second that he is indifferent, which would be true if campaigning on the same issue was as likely as campaigning on different issues.. If Challenger of this type reveals his competence in either issue, the voters' posterior beliefs about his competence will be $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. However, it is easy to see that the average competence of politicians choosing foreign policy if Incumbent chose economy is larger than this (their competence on foreign policy exceeds $\frac{1}{2}$, and that on economy exceeds $\frac{1}{4}$), and therefore, as long as $\mu < 1$, the type $(\frac{1}{2}, \frac{1}{2})$ would prefer the opposite issue. Intuitively, politicians who excel in only one dimension are more likely to be interested in revealing their competence in this dimension, whereas more symmetric politicians are less interested in communicating credibly and thus are more likely to campaign on a different issue. The difference reaches its maximum at $\mu = \frac{4}{5}$, where campaigns on different issues are almost eight percentage points (precisely, $\frac{\sqrt{3}}{3} - \frac{1}{2} \approx 0.077$) more likely than campaigns on the same issue.

Why is there no unraveling, i.e., why voters fail to interpret the politician choosing a different issue as a signal that he is the worst possible type? To see why, consider a candidate equilibrium where if Incumbent chose E , then all Challengers also choose E , with the out-of-equilibrium beliefs that if he chooses F , then his $e_c = 0$, and also $f_c = 0$, unless he credibly proves otherwise. Consider Challenger of type $(e_c = 0, f_c = 1)$. If he chooses E , he would reveal $e_c = 0$ and nothing on foreign policy, so the voters' posterior of his total competence will be $0 + \frac{1}{2} = \frac{1}{2}$. On the other hand, if he chooses F , then with probability μ he would reveal $f_c = 1$, so voters' posterior will be $1 + 0 = 1$; with probability $1 - \mu$ he will reveal nothing and the voters will assume that he is the worst type.

In expectation, the voters' posterior will equal μ . Thus, if $\mu > \frac{1}{2}$, then this type of Challenger would find it optimal to deviate and show his competence on foreign policy to the voters, because, intuitively, he has a sufficiently high chance of success. Hence, for such values of μ there is no equilibrium with unraveling; moreover, once Challengers with types close to $(0, 1)$ start to choose foreign policy, voters will Bayesian-update and think of Challengers choosing F quite highly, and then, as Proposition 2 shows, more than half of types will end up choosing F in equilibrium.

If $\mu \leq \frac{1}{2}$, equation (11) no longer defines an upward-sloping boundary, and thus the equilibrium must be different. The logic from the previous paragraph suggests that for $\mu \leq \frac{1}{2}$ there should be equilibrium with unraveling, where each Challenger chooses the same issue as incumbent. As it turns out, there are other equilibria as well.

Proposition 3 *If $\mu \leq \frac{1}{2}$, then there are multiple equilibria. Incumbent's strategy is the same ($d_i = E$ iff $e_i > f_i$) in every symmetric equilibrium. The strategy of Challenger may fall into one of the two classes:*

(i) *Challenger always conforms to Incumbent's choice of issue: $d_c = d_i$;*

(ii) *For some cutoff $t \in [\mu, 1 - \mu]$: If Incumbent chose $d_i = E$, then Challenger chooses E if $e_c > t$ and chooses F if $e_c < t$ (and for $e_c = t$, Challenger chooses E if f_c is below the cutoff $\frac{1-\mu-t}{1-2\mu}$ for $\mu < \frac{1}{2}$ and arbitrary cutoff for $\mu = \frac{1}{2}$); the strategies for $d_i = F$ are similar.*

The equilibrium strategy of Challenger may, therefore, be either to reciprocate Incumbent or to play a strategy which only depends on his competence in the issue chosen by Incumbent. If Challenger is expected to play a symmetric strategy (*mutatis mutandis*) for the two possible choices of Incumbent, the latter expects that he would be successful in communicating his competence with the same probability regardless of the issue he chooses. In this case, Incumbent will use a symmetric strategy. In principle, there exist equilibria where Challenger plays very different strategies if Incumbent chose E and F , and then Incumbent must play asymmetrically as well.¹⁴ To avoid these complications, from now on I focus on symmetric equilibria, i.e., equilibria where the equilibrium play if Incumbent and Challenger have types $(e_i, f_i, e_c, f_c) = (\alpha, \beta, \gamma, \delta)$ and $(e_i, f_i, e_c, f_c) = (\beta, \alpha, \delta, \gamma)$ will be the opposite.¹⁵

To illustrate the equilibria of type (i), suppose Incumbent chose E . If for all (e_c, f_c) , Challenger chooses E , then $\mathbb{E}(f_c | d_i = E, d(e_c, f_c) = E, e_c = x) = \frac{1}{2}$ for all x ; at the same

¹⁴For example, suppose $\mu = \frac{1}{4}$, then there is an equilibrium where all Incumbents choose E ; if that happened, Challenger also chooses E , but if Incumbent chose F , Challenger picks E if and only if $f_c < \frac{3}{4}$. In terms of Proposition 3, Challenger plays type (i) equilibrium if $d_i = E$ and type (ii) equilibrium if $d_i = F$. From Incumbent's perspective, he is able to signal his competence with probability 1 if he chooses E , and with probability $\frac{1}{4} + \frac{3}{4}\mu = \frac{7}{16} < \frac{1}{2}$ if he chooses F ; in this case it is an equilibrium for all incumbents to choose E (effectively, he has the same incentives a Challenger would have if $\mu = \frac{7}{16} < \frac{1}{2}$).

¹⁵There are many alternative ways to define the same restriction. E.g., it would be sufficient to assume that the chances of Incumbents (Challengers) with type (x, y) and with type (y, x) to be elected are the same. It is remarkable that for $\mu > \frac{1}{2}$, such symmetry need not be assumed, but may rather be proved (see Proposition 1).

time, $\hat{e}_c(y) = \mathbb{E}(e_c \mid d_i = E, d_c(e_c, f_c) = F, f_c = y)$ may be chosen arbitrarily, as may $\tilde{a}_c^F = \mathbb{E}(e_c + f_c \mid d_i = E, d_c(e_c, f_c) = F)$. If Challenger of type (x, y) chooses E , voters believe his competence is $x + \frac{1}{2}$, and if he chooses F , they believe it is $\mu(y + \hat{e}_c(y)) + (1 - \mu)\tilde{a}_c^F$. Clearly, the type most likely to deviate is $(x, y) = (0, 1)$; Challenger of this type does not deviate if and only if

$$\frac{1}{2} \geq \mu(1 + \hat{e}_c(1)) + (1 - \mu)\tilde{a}_c^F.$$

Since $\hat{e}_c(y), \tilde{a}_c^F \geq 0$, such equilibrium is possible only if $\mu \leq \frac{1}{2}$. At the same time, for such values of μ , it is indeed an equilibrium; it suffices to set $\hat{e}_c(y) = 0$ for all y and $\tilde{a}_c^F = 0$.

Equilibria of type (ii) are also possible only if $\mu \leq \frac{1}{2}$. Indeed, fix t and consider the strategies in Proposition 3 (ii). Then $\hat{f}_c(x) = \mathbb{E}(f_c \mid d_i = E, d(e_c, f_c) = E, e_c = x)$ equals $\frac{1}{2}$ if $x > t$, and may be chosen arbitrarily if $x < t$. At the same time, $\mathbb{E}(e_c \mid d_i = E, d_c(e_c, f_c) = F, f_c = y) = \frac{t}{2}$ for all y ; we also have $\mathbb{E}(e_c + f_c \mid d_i = E, d_c(e_c, f_c) = F) = \frac{t}{2} + \frac{1}{2}$. To verify whether threshold t constitutes an equilibrium, it suffices to concentrate on the types most likely to deviate. In equilibrium, type $(t + \varepsilon, 1)$ must prefer E and type $(t - \varepsilon, 0)$ must prefer F ; this yields two conditions:

$$\begin{aligned} t + \varepsilon + \frac{1}{2} &\geq \mu \left(1 + \frac{t}{2}\right) + (1 - \mu) \left(\frac{t}{2} + \frac{1}{2}\right), \\ t - \varepsilon + \hat{f}_c(t - \varepsilon) &\leq \mu \left(0 + \frac{t}{2}\right) + (1 - \mu) \left(\frac{t}{2} + \frac{1}{2}\right). \end{aligned}$$

Since these conditions need to hold for ε arbitrarily close to 0, the first condition implies $t \geq \mu$, whereas the second one (setting $\hat{f}_c(t - \varepsilon) = 0$) implies $t \leq 1 - \mu$. It is now straightforward to show that Challengers with $e_c \neq t$ do not have incentives to deviate. Showing that Challengers with $e_c = t$ have no deviations is also straightforward and is done in the Appendix.

In Subsection 3.2, voters' welfare is computed under different equilibria and parameter values, and it turns out that if $\mu \leq \frac{1}{2}$, the unraveling equilibrium, i.e., the equilibrium of type (i) dominates every equilibrium of type (ii) in terms of welfare.¹⁶ The reason for this is that the two politicians always campaign on the same issue, and a low μ does not result in loss of information due to their campaigns lacking credibility. The equilibrium of type (i) is unique for any μ , and thus this equilibrium refinement extends the uniqueness result from Proposition 1 to the case $\mu \leq \frac{1}{2}$. This is the equilibrium we focus on from now on.

3.1 Simultaneous game

Consider an alternative game, where the two politicians must choose their campaign issues simultaneously (or, equivalently, Challenger starts his campaign before he gets a chance to observe the choice of Incumbent). Consider exactly the same game as the one introduced in Section 2, except that Challenger, when deciding on d_c , does not observe the choice of Incumbent, d_i .

¹⁶The asymmetric equilibria, such as the one in Footnote 14, are also dominated by the unraveling equilibrium.

Given the symmetry of the game, it is not surprising that there is a symmetric equilibrium, where each politician j picks $d_j(e_j, f_j) = E$ if $e_j > f_j$ and $d_j(e_j, f_j) = F$ if $e_j < f_j$. Indeed, for either politician, the probability of ending up campaigning on the same issue as the opponent is exactly $\frac{1}{2}$ and this does not depend on the issue; this means that each politician is able to send a credible signal with probability $\frac{1}{2} + \frac{\mu}{2}$, regardless of the issue he chooses. Consequently, if one politician follows this symmetric strategy, then the other one also must do so in equilibrium. Hence, symmetric strategies by the politicians are “best responses” to one another, and thus such an equilibrium exists for all μ .

Proposition 4 *For any μ there exists a symmetric equilibrium where each politician $j \in \{i, c\}$ chooses $d_j(e_j, f_j) = E$ if $e_j > f_j$ and $d_j(e_j, f_j) = F$ if $e_j < f_j$. Moreover, if $\mu > \frac{1}{2}$, this is the unique equilibrium.*

This proposition does not preclude existence of equilibria which are not symmetric in the two issues: for example if $\mu \leq \frac{1}{2}$, then there is an equilibrium where $d_i(e_i, f_i) = d_c(e_c, f_c) = E$ for both politicians and all types. Indeed, Proposition 3 implies that that if Incumbent plays E , it is an equilibrium for all Challengers to pick E regardless of their type; if the moves are simultaneous, then the opposite is also true: if all Challengers are expected to pick E , it is an equilibrium for all Incumbents to do so as well.¹⁷ However, for $\mu > \frac{1}{2}$, only symmetric equilibria exist.

3.2 Social welfare

In this subsection, we study the consequences of the issue selection game on social welfare and provide comparative statics results. For the society, the relevant variable is the expected competence of the elected politician. The following lemma shows that in the probabilistic voting model as above, there is a simple formula for this expected competence. This formula applies to sequential and simultaneous games as well as some other situations (only players’ strategies matter, but not whether they form an equilibrium in a specific game played by the politicians), and we use it throughout.

Lemma 2 *Let $\mathcal{I}(e_i, f_i, e_c, f_c, \chi)$ be the voters’ posterior beliefs about the distribution of the skills of the two politicians, $(e_i, f_i), (e_c, f_c) \in \Omega_i \times \Omega_c$, that voters will get if these politicians follow the equilibrium play and χ denotes Nature’s choice to reveal competences if $d_i \neq d_c$.¹⁸ The expected*

¹⁷For $\mu \in (\frac{1}{5}, \frac{1}{2}]$, there are only three equilibria, up to measure zero: the symmetric one; all types of Incumbent and Challenger choose E , and all choose F . For $\mu \leq \frac{1}{5}$, more exotic equilibria are possible as well. For example, if $\mu \leq \frac{1}{5}$, there is an equilibrium where each politician j campaigns on E whenever $e_j \geq \frac{1}{4}$ and on F otherwise.

¹⁸This is a slight abuse of notation, as we previously used \mathcal{I} to denote the posterior distribution conditional on politicians’ choices and information obtained by voters, $\mathcal{I}(d_i, d_c, \kappa_i, \kappa_c)$. However, since in a pure strategy equilibrium, the tuple $(e_i, f_i, e_c, f_c, \chi)$ predicts $(d_i, d_c, \kappa_i, \kappa_c)$ uniquely, then $\mathcal{I}(e_i, f_i, e_c, f_c, \chi)$ is well-defined.

quality of the elected politician equals

$$1 + A \left(\sum_{\chi \in \{0,1\}} \left\{ \begin{array}{l} \mu \text{ if } \chi = 1 \\ 1 - \mu \text{ if } \chi = 0 \end{array} \right\} \int_{\Omega_i \times \Omega_c} \left(\frac{[\mathbb{E}(U(e_i, f_i) | \mathcal{I}(\dots))]^2}{+ [\mathbb{E}(U(e_c, f_c) | \mathcal{I}(\dots))]^2} \right) d\lambda - 2 \right), \quad (12)$$

where λ is the standard Lebesgue measure on $\Omega_i \times \Omega_c$.

Proof. Fix either of the realizations of χ . Following equilibrium play of politicians, the expected competence of the elected politician equals (dropping the argument at \mathcal{I} for brevity):

$$\begin{aligned} & \int_{\Omega_i \times \Omega_c} \left(\frac{\left(\frac{1}{2} + A\mathbb{E}(U(e_i, f_i) - U(e_c, f_c) | \mathcal{I})\right) \mathbb{E}(U(e_i, f_i) | \mathcal{I})}{+ \left(\frac{1}{2} - A\mathbb{E}(U(e_i, f_i) - U(e_c, f_c) | \mathcal{I})\right) \mathbb{E}(U(e_c, f_c) | \mathcal{I})} \right) d\lambda \\ &= 1 + A \int_{\Omega_i \times \Omega_c} [\mathbb{E}(U(e_i, f_i) | \mathcal{I}) - \mathbb{E}(U(e_c, f_c) | \mathcal{I})]^2 d\lambda. \end{aligned}$$

The result (12) would follow immediately (by opening the brackets and taking the weighted sum over the realizations of χ) if the expected utilities inside the last integral were independent. They are not; for example, a high realization of $\mathbb{E}(U(e_i, f_i) | \mathcal{I})$ means that most likely the politicians are talking about the same issue, and, therefore, Challenger is likely to have high average competence. However, *conditional on the choices of issues by both politicians*, these variables are independent. In addition, the choice of issue by one politician does not depend on the competence of the other one, except through the choice of issue, even in the case of sequential moves. This implies that social welfare may be computed using (12). Appendix A fills in the details. ■

Lemma 2 simplifies the computations considerably. In particular, it shows that the expected competence of the elected politician only depends on the sum of variances of posterior beliefs that politicians' equilibrium play generates. This allows to do the computations for Incumbent and Challenger separately, only taking into account the equilibrium strategies. Interestingly, formula (12) shows that voters' welfare increases in the variance of their posterior beliefs about each politician's total competence. This is intuitive; it means that a more informative campaign, which generates more heterogeneous beliefs, results in higher welfare than a less informative one, where posteriors might be not that different from the priors.

In evaluating the welfare consequences, the following benchmarks are useful. First, if the winner were picked at random, the average competence would be the unconditional expectation, i.e., 1. At the opposite extreme is the full information case: if both candidates revealed their competences to voters on both dimensions, then the expected quality of the winner would be

$$1 + A \left(2 \int_0^1 \int_0^1 (x + y)^2 dx dy - 2 \right) = 1 + \frac{1}{3}A. \quad (13)$$

Thus, $\frac{1}{3}A$ is the extra benefit of having elections as opposed to picking the candidate randomly; this is the maximum one can achieve with probabilistic voting where a less competent candidate

has a chance due to a shock to preferences θ . As the variance of the common shock decreases (A becomes higher), the expected competence of the elected politician would increase.

In the case of sequential voting, we have the following result.

Proposition 5 *The expected competence of the elected politician is increasing in A . It is non-monotone in μ ; more precisely, it equals $1 + \frac{5}{24}A$ if $\mu \leq \frac{1}{2}$ (if type (i) equilibrium from Proposition 3, which yields the highest social welfare among all equilibria, is played), and for $\mu > \frac{1}{2}$, it monotonically increases from $1 + \frac{3}{16}A < 1 + \frac{5}{24}A$ to $1 + \frac{1}{4}A > 1 + \frac{5}{24}A$.*

The nonmonotonicity result is not surprising if one takes into account that for $\mu < \frac{1}{2}$, the two politicians are guaranteed to discuss the same issue (in the welfare-maximizing equilibrium), whereas for μ slightly exceeding $\frac{1}{2}$ such equilibrium does not exist, and the politicians will talk about different issues at least half of the time (even more than that, as follows from Proposition 2), and thus there is a chance of about one-fourth that they will fail to announce their respective competences. In fact, the expected competence exceeds $1 + \frac{5}{24}A$ only if $\mu > 0.7$. In all cases, this falls short of the maximal possible gain of $\frac{1}{3}A$, although if μ is close to 1, then 75% of this gain is realized, and even in the worst-case scenario this chance exceeds 56% ($\frac{9}{16}$).

Consider now the welfare implications of a game where strategies are chosen simultaneously. Proposition 4 established that a symmetric equilibrium exists for all μ , and arguably it is the most plausible one (and unique if $\mu > \frac{1}{2}$). We get the following comparison in this case.

Proposition 6 *In the symmetric equilibrium of the game with simultaneous moves, the expected quality of the elected politician equals $1 + \frac{1+\mu}{8}A$; this is lower than the expected quality in the game of sequential moves if $\mu \leq \frac{1}{2}$, but it is higher than that if $\frac{1}{2} < \mu < 1$.*

The intuition for this result is simple: all things equal, voters make a more informed choice, and therefore get a higher utility, if politicians reveal more information about their competences. This is more likely to happen (again, all things equal) if μ is high, and given that the strategies are the same, the expected quality is increasing in μ . The probability of campaigning on the same issue in the game with simultaneous moves is $\frac{1}{2}$; on the other hand, Proposition 2 states that in the game with sequential moves, it is less than $\frac{1}{2}$ for $\mu \in (\frac{1}{2}, 1)$. This explains higher voter welfare for such μ in simultaneous game. If $\mu \leq \frac{1}{2}$, then sequential moves allow politicians to converge on the same issue and have an informative discussion at least on that issue. With simultaneous moves, they lack the ability to converge, and thus the welfare of voters is lower in this case.

If there is a concern that social welfare is lower if politicians fail to coordinate on the same issue, then one alternative would be to choose an issue (say, at random), and then require that both politicians campaign on that issue. Formula (12) applies to this case as well, and the expected

quality of the winner equals

$$1 + A \left(2 \int_0^1 \int_0^1 \left(x + \frac{1}{2} \right)^2 dx dy - 2 \right) = 1 + \frac{1}{6} A \quad (14)$$

(indeed, if a politician announces competence x on one dimension, the voters expect his total competence to be $x + \frac{1}{2}$). Comparing this result to Proposition 5 reveals that for all μ , the expected competence in the case where politicians are free to choose their issue is higher than if they are not. This seems surprising, because if the issue is fixed, there is no loss due to possible failure to announce their competences credibly in any dimension. However, there is a different force at play: with endogenous choice of issues, the announcement of a politician of his competence over one issue carries quite a bit of information about his competence on the other issue, which is not the case if they were forced to talk on a given issue (this is true for Incumbent for any μ , and for Challenger if $\mu > \frac{1}{2}$). It turns out that the latter effect dominates.¹⁹

Proposition 7 *If politicians were forced to campaign on an exogenously chosen dimension, then for all values of μ , social welfare would be strictly lower than in the game of sequential moves. It would be lower than in the game with simultaneous moves as well (if the symmetric equilibrium is played), provided that $\mu > \frac{1}{3}$; if, however, $\mu < \frac{1}{3}$, then forcing politicians to campaign on an exogenously given dimension results in a higher social welfare than a simultaneous-move game.*

Figure 3 illustrates the expected qualities of elected politicians under different scenarios. Proposition 7 suggests that the only scenario where allowing politicians to choose issues hurts welfare is when this choice is simultaneous (so the politicians do not coordinate), and also μ is sufficiently low, so voters often fail to get precise information about candidates' competences.²⁰ This leads to the following nontrivial observation. When candidates choose an agenda for an entire campaign, the decisions are unlikely to be made at once, and hence constraining the candidates by imposing an issue on them is not a good idea. However, when it comes to some particular event, such as a debate, where candidates are likely to prepare their communication strategy without observing the opponent's plan, fixing a particular issue may make sense. Notice that for $\mu \in (\frac{1}{2}, 1)$, it is better for voters if politicians made their campaign decisions independently; indeed, with sequential decisions,

¹⁹In Subsection 5.2, we consider the case where issues have different weights. It turns out that allowing politicians to choose issues freely is still optimal: even though the voters would prefer that the politicians compete on the more important issue, they would do that in equilibrium anyway. We show this result formally for $\mu = 1$; in the sequential game, numerical simulations confirm this hypothesis for other values of μ as well. However, for more general distributions of types this is not necessarily true: for example, if the competence of Challenger over one dimension is very uncertain and $\mu < 1$, the society would strongly prefer Incumbent to choose that issue, but Incumbent would ordinarily do that with probability 1/2 only.

²⁰Caselli and Morelli (2004) and Mattozzi and Merlo (2007) consider very different models that lead to selection of incompetent politicians. In this paper, incompetent politicians may get elected because voters do not necessarily make a strong inference about a politician's incompetence in the issue he is not campaigning on.

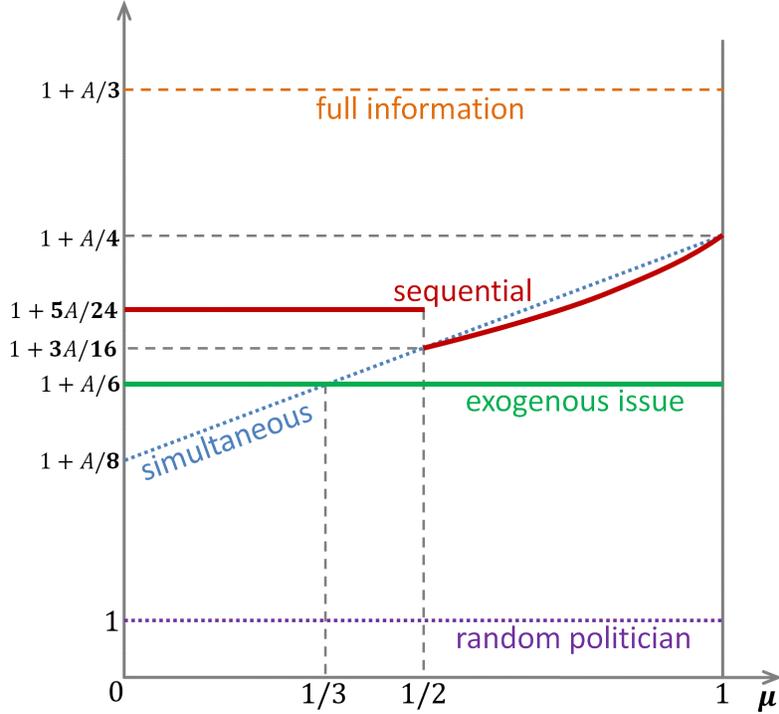


Figure 3: Social welfare under different scenarios and values of μ .

it is more likely that they will campaign on different issues, while with independent decisions this probability is fixed at $\frac{1}{2}$.

3.3 Probability of being elected

Another natural question is whether the timing of the game gives an advantage to Incumbent or Challenger. At first glance, the sequential timing allows Incumbent to pick the issue that he prefers, and thus can guarantee that he will talk about the issue that he excels at. However, the voters are aware of this strategy, and discount Incumbent's competence on the other issue accordingly. Challenger, on the other hand, does not have such flexibility, and if $\mu < \frac{1}{2}$ he is compelled to choose the same issue as Incumbent, which might not be his strong side. The voters understand this, and do not infer Challenger's competence in the other dimension from his announcement, and thus if Challenger turns out to be incompetent in the issue that Incumbent picked, he is not penalized further. In other words, Incumbent has a higher chance to establish his high competence, but if he fails to do so, he will be perceived as very incompetent; in contrast, the variance of voters' posterior beliefs about Challenger is likely to be smaller. In the probabilistic voting model, however, it is the voters' posterior expectations of the politicians' competences that matter, and they are equal for the two politicians as long as they are taken from the same distribution. This implies, for example, that before knowing his type, a politician is indifferent between being the first-mover and

the second-mover in the game.

Proposition 8 *The probability of Incumbent winning and Challenger winning are equal.*

At the same time, a given type of politician need not be indifferent. For example, if $\mu < 1$, then all politicians who are equally competent on both dimensions, with $e_j = f_j < 1$, would prefer to be second-movers rather than first-movers. In Subsection 5.4, Incumbent will be given an option to postpone his announcement and allow Challenger to move first. It turns out that while not all types of Incumbent will use this option, those who do are likely to be competent, and this will result in observable first-mover advantage.²¹

4 Dynamics of campaign

In this Section, I extend the baseline model by allowing the Incumbent (who moves first) to reconsider his initial choice if the Challenger chose a different issue. The idea here is to capture the process of finding a common theme for the campaign or debates and to make sure that both players have an opportunity to respond to the opponent’s suggestion. Specifically, consider the following timing. As before, Incumbent moves first and picks a (now tentative) issue for his campaign, \tilde{d}_i . Challenger observes this choice and responds with his own (final) decision d_c . If the two issues coincide, $\tilde{d}_i = d_c$, then the parties proceed to campaigning on this issue, and so Incumbent’s final decision is $d_i = \tilde{d}_i$. If, however, $\tilde{d}_i \neq d_c$, then with some probability p , Incumbent has an opportunity to revise his initial decision, and is free to pick any $d_i \in \{\tilde{d}_i, d_c\} = \{E, F\}$, and with a complementary probability $1 - p$ he has to stick to $d_i = \tilde{d}_i$. The probability p may correspond to the chance that the Incumbent’s campaign team is flexible enough or has enough time to switch the focus.²² This timing (as well as a possible conversation) is shown on Figure 4.²³ I assume that voters observe the entire sequence of moves (\tilde{d}_i, d_c, d_i) ; in particular, they know whether Incumbent was the first to propose the issue of his campaign d_i , or he started with a different issue and then switched. In what follows, restrict attention to symmetric equilibria, which implies, in particular, that the first proposal by Incumbent is $\tilde{d}_i = E$ if $e_i > f_i$ and $\tilde{d}_i = F$ otherwise. To ensure existence

²¹In Ashworth and Bueno de Mesquita (2008), incumbency advantage arises in a probabilistic voting model due to his ex-ante higher competence (which in turn is present because he had won elections before). Since this paper assumes, for simplicity, that the candidates are ex-ante symmetric, this effect is not present.

²²Alternatively, one can think of p as the share of voters who get Incumbent’s message after he switches, and $1 - p$ is then the share of voters who pay attention at the beginning of the campaign only. Probabilistic voting ensures that these two interpretations are equivalent.

²³If Incumbent were allowed to revise his initial pick even if Challenger chose the same issue, then the first-stage announcement by Incumbent would have to involve “babbling”. The reason is that in a candidate equilibrium where first announcement is informative, switching to another issue should be interpreted as competence in both issues. Thus, it would make sense to announce the weaker issue for at least some types, and this cannot be true in equilibrium. I am indebted to V. Bhaskar for the suggestion to explore the possibility that Incumbent must stick to his original choice if Challenger approves the choice of issue.

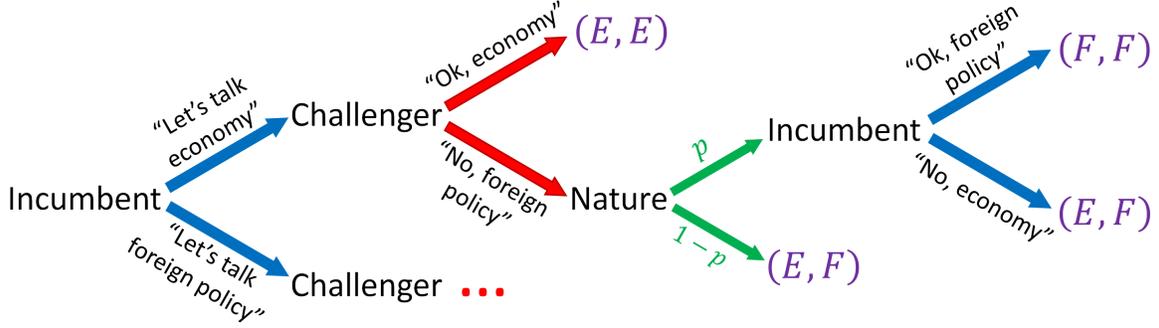


Figure 4: Timing of proposal-making by Incumbent and Challenger in the dynamic game.

of such equilibria, assume that p is not too close to 1 (precisely, we need $p \leq 4\sqrt{3} - 6 \approx 0.93$);²⁴ to avoid issues with multiplicity, assume $\mu > \frac{1}{2}$.

Let us focus on Incumbent's second choice on whether to switch the campaign issue; this question is relevant only if $d_c \neq \tilde{d}_i$. For concreteness, suppose $\tilde{d}_i = E$ and $d_c = F$. In this case, Incumbent may either agree to discuss foreign policy or insist on talking about economy. Suppose first that Incumbent is very competent in economy but not in foreign policy, for example, his type is $(1, 0)$. For such Incumbent, switching to foreign policy is unlikely to do him any good: indeed, he would have a (weakly) higher chance to signal his competence credibly, but the signal will be about his much weaker dimension, foreign policy, and this is precisely what he prefers to avoid. In contrast, suppose that Incumbent is very competent in both dimensions, say, type $(1, 1 - \varepsilon)$. In this case, it makes perfect sense to comply with Challenger's proposal and switch to foreign policy. By doing so, not only he will be able to signal that he is highly competent in foreign policy; in addition, voters will take into account that foreign policy is his weaker issue, and therefore he must be even more competent in economy (which he truly is). For such Incumbent, therefore, switching allows to signal competence in both dimensions, which is otherwise difficult to achieve. If he insisted in talking about economy, he would, with some probability, reveal his highest competence in this issue, but voters would view his foreign policy credentials to be average at best (and, in fact, worse than average, because they expect those who excel in foreign policy to switch). Therefore, one can expect that in equilibrium Incumbent will insist on campaigning on the issue he originally chose if

²⁴The reason why existence may fail if p is too close to 1 is the following. Consider Incumbent who is very competent in both issues, but slightly better at E (e.g., type $(1, 1 - \varepsilon)$), and suppose he chooses $\tilde{d}_i = E$ at first. From Proposition 2, we know that if $\mu > \frac{1}{2}$, Challenger will pick $d_c = F$ with probability higher than $\frac{1}{2}$; it turns out that this remains true in this version of the game as well (in fact, the possibility of Incumbent switching increases Challenger's chance of conveying his competence, and increases the 'effective μ '. But this would imply that very competent Incumbents who, as we will see, are always willing to switch to signal their competence on both issues, expect to end up talking about F with a higher probability than about E . But if he switched to F , he would, similarly, end up campaigning on E with a higher probability, which he prefers. This leads to nonexistence of a monotone equilibrium in pure strategies. On the other hand, if $p \leq 4\sqrt{3} - 6$, then the probability of talking about E upon choosing $\tilde{d}_i = E$ equals $(1 - p) + pz \geq \frac{1}{2}$, which ensures that monotone strategies form an equilibrium (here, $z = \frac{1}{2} - \frac{1}{2} \left(\frac{\sqrt{3}}{3} - \frac{1}{2} \right) = \frac{3}{4} - \frac{1}{6}\sqrt{3} \approx 0.46$ is the smallest probability that Challenger will choose the same issue as Incumbent).

his competence in the other issue is low, but will be open to switching if he is competent on the other issue as well.

What is the choice of Challenger who received a proposal to talk about, say, economy, and anticipates that if he proposes foreign policy instead, then Incumbent will follow the strategy above? Such Challenger knows that if he agrees on E , then both will campaign on economy, and his signal will be credible with probability 1. At the same time, if he chooses F instead, he will talk about F , but will only be able to send a credible signal with probability $\mu' = p\eta + ((1-p) + p(1-\eta))\mu = \mu + p\eta(1-\mu)$, where η is the probability of Incumbent switching to F if given such chance. Since we assumed $\mu > \frac{1}{2}$, we are guaranteed to have $\mu' \geq \mu > \frac{1}{2}$, and therefore the characterization of Challenger's strategies from Proposition 1 applies (with μ replaced by μ'). It remains to verify that Incumbent's strategy to start with the issue he is most competent in is indeed an equilibrium; one can verify that this is true if p is not too close to 1, as we assumed. We therefore have the following result, which is illustrated in Figure 5.²⁵

Proposition 9 *Suppose that $p < 2(2\sqrt{3} - 3) \approx 0.93$. If $\mu > \frac{1}{2}$, there is a unique symmetric equilibrium. In this equilibrium, Incumbent initially chooses the issue that he is more competent at, and if Challenger picks a different issue, then a positive share of Incumbents switch. The boundary between those willing to switch and those that are not is linear; for $\mu = 1$, Incumbent is flexible if $\min(e_i, f_i) > \frac{1}{2} \max(e_i, f_i)$, i.e., if Incumbent is relatively symmetric; at μ close to $\frac{1}{2}$, Incumbent switches if $\min(e_i, f_i) > \frac{4-\sqrt{10}}{3} \approx 0.28$. Challenger follows the strategy described in Proposition 1 for some $\mu' \geq \mu$.*

This means that the types who switch issues in the course of the campaign are, on average, more competent and well-rounded, whereas the types who do not are more asymmetric in their competence, and less competent in general. As a result, the types who switched are also more likely to be elected. This is consistent with anecdotal evidence: in 2012, Obama switched the focus of his campaign, thus establishing his credentials in several issues, and won the election, even though a majority of voters thought Romney would do a better job on what they considered the most important issue, the economy.²⁶

²⁵The politician's decision to stick to the original choice or switch would largely be the same in a more symmetric variant of the game, where Incumbent and Challenger choose their issues simultaneously at first, and if they chose different issues, then one of them (say, with probability $\frac{1}{2}$ each) gets a chance to revise.

²⁶According to CBS news (<http://www.cbsnews.com/news/early-exit-poll-60-percent-say-economy-top-issue/>), "Sixty percent of voters who cast ballots on Election Day or earlier say the economy is the most important issue in their vote, according to an early CBS News exit poll.

In CBS News/New York Times poll of likely voters taken shortly before the election, Mitt Romney had the edge over President Obama on the question of which candidate would do a better job handling the economy, 51 percent to 45 percent."

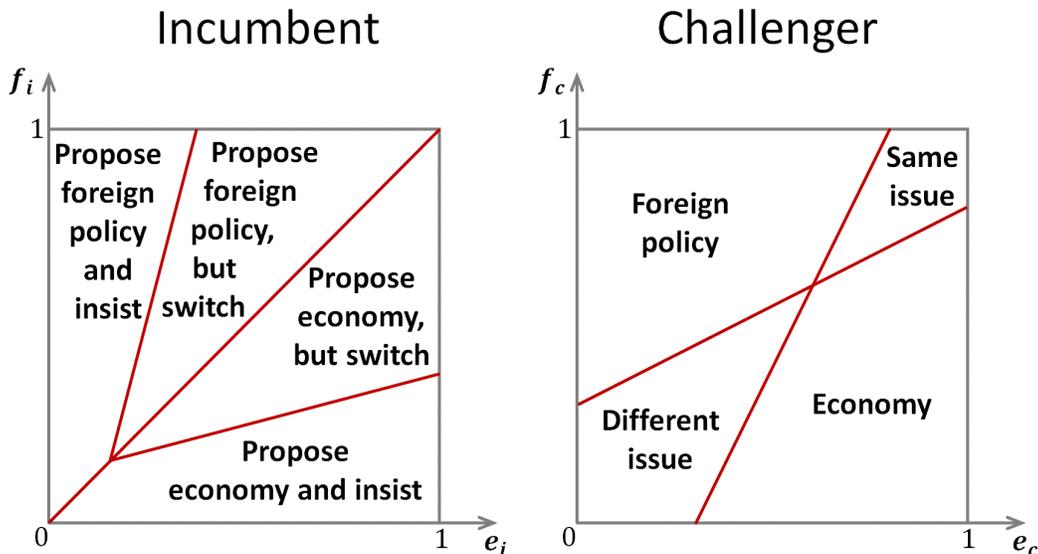


Figure 5: Equilibrium strategies by Incumbent and Challenger in the dynamic game if $\mu > \frac{1}{2}$.

5 Extensions

In this section, I consider several extensions of the baseline model of Section 2.

5.1 Asymmetric uncertainty

The baseline model assumed that the ex-ante distributions of Incumbent and Challenger's abilities are the same. This was obviously a simplification; first, the voters are likely to be better informed about Incumbent's competence, and also Incumbent is more likely to be more competent on the grounds that he was selected into office earlier. Suppose that, from voters' perspective, Incumbent's two-dimensional type is taken not from a uniform distribution on $[0, 1] \times [0, 1]$, but instead from a uniform distribution on $\Omega'_i = [e_1, e_2] \times [f_1, f_2]$; in particular, $e_1 = e_2$ would imply that the voters are perfectly informed about Incumbent's ability on economy, and $f_1 = f_2$ would imply the same on foreign policy. The new distribution of Incumbent's type is shown in Figure 6.

The Challenger's strategies, for either choice made by Incumbent, are the same as specified in Proposition 1 for $\mu > \frac{1}{2}$ and Proposition 3 for $\mu \leq \frac{1}{2}$. The strategy of Incumbent, as it turns out, critically depends on the shape of the set Ω'_i . In particular, if it is a square, albeit smaller than $\Omega_i = [0, 1] \times [0, 1]$, which means that the residual uncertainty of Incumbent's competence in the two issues is the same, then Incumbent will have equal probability of campaigning on both issues. Interestingly, this does not depend on whether he is known to be competent or incompetent in either dimension; all that matters is residual uncertainty. If, however, uncertainty about Incumbent's competence in one of the dimensions is less, then Incumbent is more likely to campaign on the other dimension (and in particular, the most competent and the least competent of Incumbent's

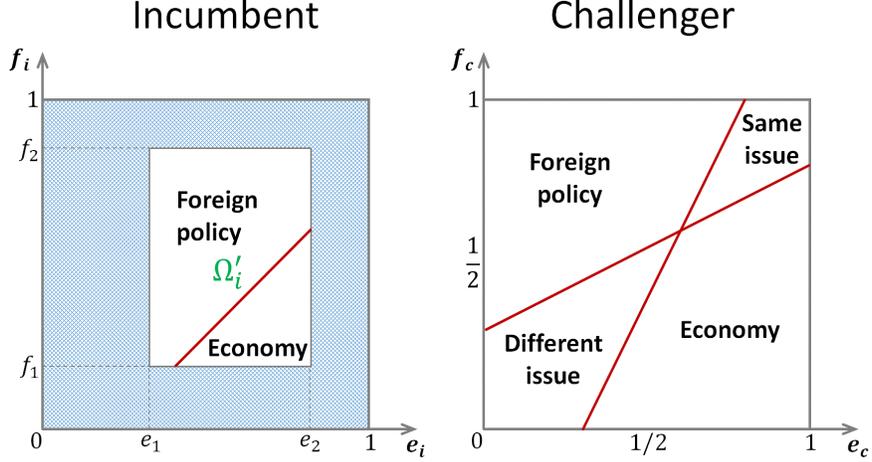


Figure 6: Information structure and equilibrium strategies.

types will choose the other dimension, provided that μ is not equal to 1). More formally, we have the following result.

Proposition 10 *Suppose that Incumbent's type is distributed on $\Omega'_i = [e_1, e_2] \times [f_1, f_2]$. Then:*

(i) *If $e_2 - e_1 = f_2 - f_1$, then Incumbent is equally likely to choose either dimension, and will choose E if $e_i - e_1 > f_i - f_1$ and will choose F if $e_i - e_1 < f_i - f_1$.*

(ii) *If $e_2 - e_1 < f_2 - f_1$, then in any equilibrium, Incumbent is more likely to choose F than E. More precisely, let ξ be Incumbent's probability of being credible in communication.²⁷ Then for $\frac{e_2 - e_1}{f_2 - f_1} < 1 - \xi$, there is a unique equilibrium, in which all types of Incumbent will choose F; if $\frac{e_2 - e_1}{f_2 - f_1} \in \left(\frac{1 - \xi}{2\xi - 1}, 1\right)$, there is a unique equilibrium where Incumbent chooses both E and F with positive probabilities; and for $\frac{e_2 - e_1}{f_2 - f_1} \in \left[1 - \xi, \frac{1 - \xi}{2\xi - 1}\right]$, there are two equilibria. A similar characterization applies if $e_2 - e_1 > f_2 - f_1$.*

(iii) *The probability that Incumbent chooses E increases as the ratio $\frac{e_2 - e_1}{f_2 - f_1}$ increases.*

In other words, Incumbent is always more likely to campaign on the issue where voters are more uncertain about his competence. At the extreme, if voters perfectly know his competence on either of the dimensions, Incumbent must campaign on a different issue. Interestingly, this perfect knowledge is not required for this result. If $\mu < 1$, so there is a chance that voters will fail to get the precise signal, then Incumbent will never choose economy if $\frac{e_2 - e_1}{f_2 - f_1}$ is small enough, and will never choose foreign policy if $\frac{f_2 - f_1}{e_2 - e_1}$ is small enough. Intuitively, if $e_2 - e_1$ is small and $f_2 - f_1$ is not, then even Incumbent with $e_i = e_2$ will not want to waste his campaign on the issue of economy, where the campaign cannot make a big difference, and will talk about foreign policy instead. Thus, one can expect that an Incumbent who had a chance to demonstrate his true competence (or true

²⁷This probability ξ is the same for both issues, and equals $\xi = \pi + \mu(1 - \pi)$, where π is the probability that Challenger chooses the same issue as Incumbent.

incompetence) in one of the issues during his term or career prior to the campaign will build his campaign on a different issue.

5.2 Asymmetric issues

We have assumed so far that voters care about both issues equally. A natural question is what will happen, both to campaign strategies and social welfare, if voters weighted the two issues differently. Suppose that voters have the following utility function:

$$U(e, f) = w_e e + w_f f = (1 + \Delta) e + (1 - \Delta) f; \quad (15)$$

then the baseline case (1) would correspond to $\Delta = 0$.

Assume, as before, that both politicians are taken from a uniform distribution on $[0, 1]^2$. In what follows, we restrict attention to the case where $\mu = 1$; this case allows for simple closed-form solutions and has the additional advantage that the politicians' strategies do not depend on whether the game is sequential or simultaneous. We should now expect politicians to have a preference for revealing their competence along the dimension that voters care more about. This is similar to Subsection 5.1, where competent politicians had an incentive to inform voters on the issue where there is more uncertainty; here, by informing voters about his competence on an issue they do not care about, a politician barely changes their propensity to vote for him. We should therefore expect that politicians will be more likely to campaign on the more important issue, and this is indeed what happens in equilibrium.

We start by characterizing the equilibrium strategies.

Proposition 11 *Suppose that voters' utilities are given by (15). Then in both simultaneous and sequential games, a politician with type (e, f) will choose E if and only if:*

$$f < \frac{w_e}{w_f} e = \frac{1 + \Delta}{1 - \Delta} e.$$

Moreover, the voters' expected utility is strictly greater in this case than if the politicians had to compete on one issue chosen by voters (and also is increasing in $|\Delta|$).

The equilibria are depicted in Figure 7. Proposition 11 implies that politicians are indeed more likely to compete on the more important issue. This effect is sufficiently strong to have welfare consequences. A priori, one could think that it would be better to require the politicians to campaign on the more important issue: even though there is no direct information loss since $\mu = 1$, the possibility that a politician reveals his competence on the less important issue is wasteful for voters. However, the more important one issue is, the less frequently politicians will campaign on the other issue, and this effect is strong enough so that the voters would be worse off if they fixed campaigns on the more important issue (and they would be even more worse off if they fixed the less important one).

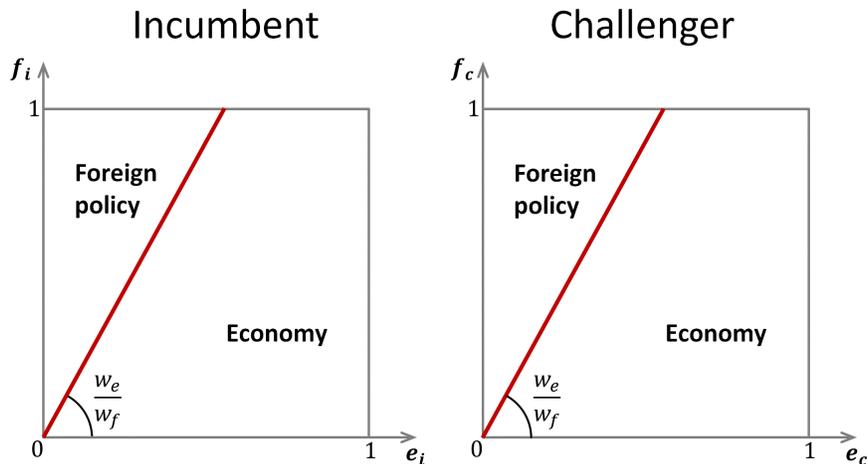


Figure 7: Equilibrium strategies if E and F have different weights, and $\mu = 1$.

5.3 Campaigning on more than one issue

So far, the politicians faced a hard constraint that they must choose one issue to campaign on. One feature of political campaigns, however, is that politicians can spend money to buy voters' attention. This possibility may be easily incorporated into this framework.

Suppose that each of the politicians can freely campaign on one issue, however, by paying a certain amount of money M , he can campaign on both issues. Thus, Incumbent chooses $d_i \in \{E, F, EF\}$ and Challenger chooses $d_c \in \{E, F, EF\}$. As before, voters observe the issues that the politicians campaign on, but may or may not learn the exact competences of politicians on those issues. If Incumbent campaigns on E (i.e., $d_i \in \{E, EF\}$), then voters learn e_i with certainty if Challenger also covers E (i.e., if $d_c \in \{E, EF\}$), and with probability μ if he does not (i.e., if $d_c = F$); the information structures for foreign policy and for Challenger's campaigns are similar. In other words, the only new assumption is that information obtained by voters on a politician's competence on one issue is not directly affected by his choice to campaign on the other issue.

To build an intuition, consider a simultaneous-move game and suppose that $\mu = 1$. If campaigning on both issues were impossible, each politician would choose his stronger issue. Now take one of the politicians, say Incumbent, and suppose that his stronger issue is E ; naturally, he would be more inclined to talk about both issues if he is good at F as well. Suppose his type is (e, f) and he is indifferent between talking about both issues and E only. If he chooses both issues, voters learn his true competence $e + f$; if he chooses E only, their posterior will in equilibrium be $e + \frac{f}{2}$. He is thus indifferent if and only if $A \left(e + f - e - \frac{f}{2} \right) = M$, i.e., if $f = \frac{2M}{A}$. Thus, a lower M makes campaigning on both issues more likely; this strategy will be chosen by politicians that are relatively competent in both issues, while the less competent ones will choose their best issue only. More generally, the following is true for the simultaneous-move game if players play symmetric

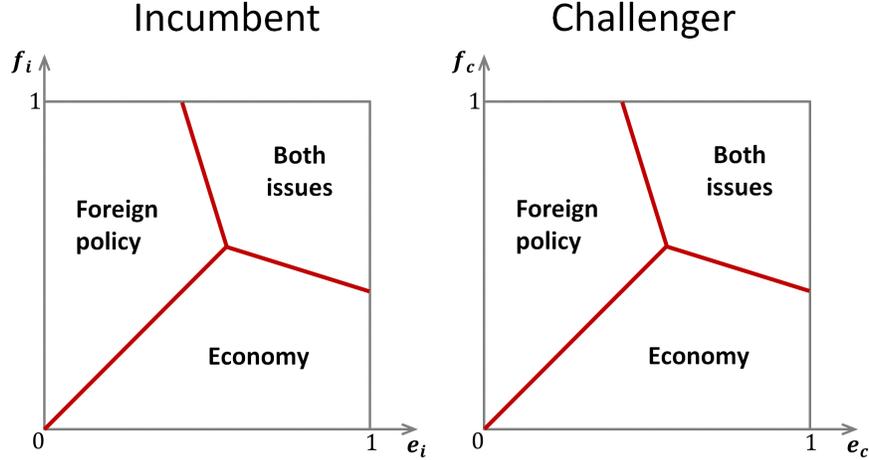


Figure 8: Equilibrium strategies in simultaneous-move game if campaigning on both issues is allowed at a cost, and $\mu < 1$.

equilibria.

Proposition 12 *Take a simultaneous-move game and fix any $\mu > 0$. Then if $M > A^{\frac{3-\mu}{4}}$, politicians always talk about one issue. For $M \in \left(0, A^{\frac{3-\mu}{4}}\right)$, politicians choose both issues with a positive probability, which is increasing in M . The politicians who choose only one issue choose their best issue. Finally, if $M = 0$, then almost all politicians campaign on both issues.*

The equilibrium strategies are depicted in Figure 8. Quite interestingly, if $\mu \in (0, 1)$, the lines that separate politicians talking about both issues from those talking about only one issue are downward sloping. This is surprising at first: for someone who is extremely good in one issue, it might suffice to talk about that issue only. However, the intuition is very simple. Recall that in equilibrium of the original game where politicians could only talk about one issue, those who are revealed to be bad at the issue they chose (say, economy) also leave little uncertainty about the other issue, and all but prove that they are even worse on that dimension. In contrast, those who demonstrate that they are extremely good at economy leave a high degree of uncertainty about their competence in the other dimension. Thus, politicians that are better at one issue have more uncertainty to resolve and thus have more incentives to campaign on both issues.

What are the welfare consequences of the possibility to campaign on both issues, at a cost? To answer this question, focus on the case where $\mu = 1$ where the equilibrium strategies are particularly simple (also, in this case, both sequential and simultaneous games yield identical results).

Proposition 13 *Suppose that $\mu = 1$. Then in both simultaneous and sequential games, if $M > \frac{A}{2}$, politicians always talk about one issue; type (e, f) chooses E if and only if $e > f$. If $M \in \left(0, \frac{A}{2}\right)$, then politician chooses both issues if $\min(e, f) > \frac{2M}{A}$, and otherwise chooses e if and only if $e > f$.*

The voters' welfare is decreasing in M . The politicians' joint expected welfare is nonmonotone, with two global maxima at the ends of the interval $[0, \frac{A}{2}]$, and a global minimum at $M = \frac{A}{6}$ (for that value, $\frac{4}{9}$ of politicians cover both issues in their campaigns). The total weighted welfare of voters and politicians is decreasing in M if politicians' weight is low relative to that of voters' and is nonmonotone otherwise.

The result on social welfare is not surprising. Since a lower M makes more politicians willing to campaign on both dimensions, campaigns are more informative, and voters are unambiguously better off. For politicians, election itself is a zero-sum game, and they would ex ante prefer not to make additional expenditures. Thus, their utility is maximized when either campaigning on both issues is costless ($M = 0$) or does not happen in equilibrium ($M \geq \frac{A}{2}$). Not surprisingly, the society as a whole would benefit from more transparency and more informed voters. An extension where voters' attention, which ultimately determines M , is endogenized, is an interesting direction for future research.

5.4 Delaying campaign

So far, Incumbent was assumed to be the first mover and Challenger was assumed to follow. Suppose, however, that Incumbent has a choice whether to move first or wait for Challenger to start his campaign and then move second. Proposition 8 suggests that on average, first mover and second mover have equal chances of winning. However, Incumbent who knows his type (e_i, f_i) need not be indifferent. For simplicity, let us focus on equilibria where the second mover chooses the same issue as the first mover; to ensure existence of such equilibria, restrict attention to the case $\mu \leq \frac{1}{2}$.

To build intuition, let us first figure out Incumbent's preferences to be the first or second mover. Consider Incumbent of type (e_i, f_i) and suppose that $e_i \geq f_i$. If Incumbent moves first, the voters' posterior will be $e_i + \frac{e_i}{2} = \frac{3}{2}e_i$, because Challenger will choose the same issue (Economy). If he moves second, then with probability $\frac{1}{2}$ Challenger will choose E , and then Incumbent's perceived competence will be $e_i + \frac{1}{2}$ (since all Incumbents will choose E in this case), and with probability $\frac{1}{2}$ Challenger will choose F , in which case Incumbent's perceived competence will be $f_i + \frac{1}{2}$. Incumbent prefers to move first if and only if $\frac{3}{2}e_i > \frac{1}{2}(e_i + \frac{1}{2} + f_i + \frac{1}{2})$, i.e., if $f_i > 2e_i - 1$, and to move second otherwise. Similarly, if $e_i \leq f_i$, then Incumbent prefers to move first if $e_i > 2f_i - 1$. This is illustrated on Figure 9 (Left). In other words, Incumbents that are competent in one issue and incompetent in the other prefer to move first, whereas those who prefer to move second tend to be, on average, well rounded and generally incompetent. Intuitively, politicians with types close to $(1, 0)$ or $(0, 1)$ have strong preferences over issues to campaign on and want to be the ones who choose the issue. On the other hand candidates with $e_i = f_i < 1$ strictly prefer to be second movers:

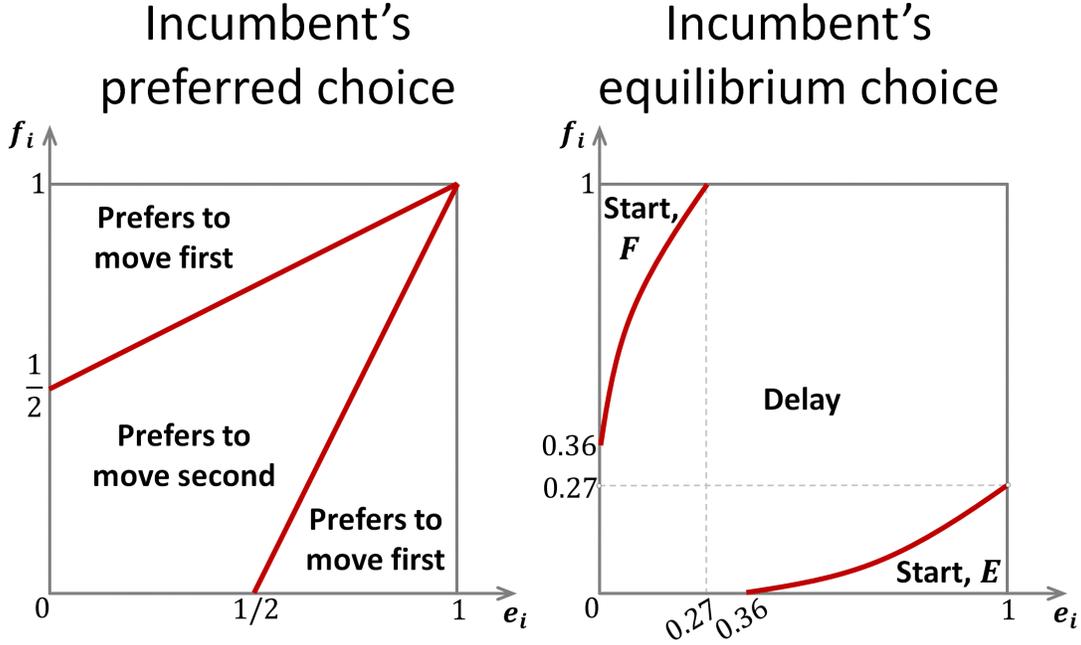


Figure 9: Left: Incumbent's preferred choice whether to move first or second. Right: Incumbent's equilibrium choice for parameters $l_1 = l_2 = \frac{1}{20}$. The exact values are $\frac{\sqrt{161}-3}{36} \approx 0.27$ and $\frac{\sqrt{157}+7}{54} \approx 0.36$.

they know that if they move first, their competence on the issue they do not campaign on will be heavily discounted, and this is less of a problem if they move second. Indeed, in the latter case, the voters will think that they were forced to campaign on a dimension chosen by Challenger, and will not penalize them. The least competent politicians (in particular, those with $e_i, f_i < \frac{1}{2}$) prefer to move second as well: for them, moving first and campaigning on either issue is going to release a (justified) negative signal about their competence in both dimensions; at the same time, if they move second, they would reveal their incompetence in one issue only.

Let us now use this intuition to analyze the game where Incumbent, in the beginning of the game, makes a strategic choice whether to move first or wait and move second, and voters observe this choice. It is easy to see that in that case, the strategies from Figure 9 (Left) cannot be equilibrium. Indeed, take Incumbent of type $(e_i, f_i) = (\frac{3}{4}, \frac{1}{2})$, who was indifferent between moving first or second. Now that voters understand that moving first or second is a strategic decision, he is no longer indifferent: moving first (and choosing E) will lead to voters' posterior of $\frac{3}{4} + \frac{1}{2} (\frac{1}{2} + 0) = 1$ (because voters will take the expectation of his foreign policy skills among Incumbents who preferred to move first and choose E), whereas moving second will give him $\frac{3}{4} + \frac{1}{2} (\frac{7}{8} + \frac{1}{2}) = \frac{23}{16}$ if Challenger chooses E and $\frac{1}{2} + \frac{1}{2} (\frac{3}{4} + 0) = \frac{7}{8}$ if Challenger chooses F . Thus, waiting gives him, in expectation, $\frac{37}{32} > 1$, which makes him strictly prefer to wait. This is intuitive: moving first allows them to reveal their competence in the issue they are good at, but now voters understand that their impatience

means that they are very incompetent in the other issue. On the other hand, waiting signals to voters their skills in the two issues are relatively close, and therefore announcing their competence in one issue will not hurt them on the other dimension, or will hurt very little. Hence, in equilibrium, Incumbents will have a strong incentive to wait to signal their symmetry.

To describe equilibrium, consider a game where with probability $l_1 > 0$ Incumbent must move first, with probability $l_2 > 0$ he must move second, and with complementary probability $1 - l_1 - l_2$ he has discretion when to move. The inference problem that voters face is now more complicated: not only they need to take into account two possibilities (whether Incumbent moved first or second because he had to or because he chose to do so), but also the relative probabilities of these two scenarios depend, in a nontrivial way, on the competence that Incumbent demonstrated in the dimension he campaigned on. This makes Bayesian updating quite a bit more involved and, unlike all previous cases in the paper, the boundaries separating types of Incumbents who move first and who move second will be nonlinear. While a comprehensive analysis of this game is beyond the scope of this paper, we can prove the following result.

Proposition 14 *Suppose $l_1 + l_2 \leq \frac{1}{10}$ and $\mu \leq \frac{1}{6}$. Then there is an equilibrium with the following properties:*

(i) *Incumbent with $e_i \geq f_i$ moves first if and only if*

$$e_i + \frac{1}{2} \frac{l_1 e_i^2 + (1 - l_1 - l_2) f_i^2}{l_1 e_i + (1 - l_1 - l_2) f_i} > \frac{1}{2} \left(e_i + \frac{1}{2} \frac{l_2 + (1 - l_1 - l_2) (1 - f_i^2)}{l_2 + (1 - l_1 - l_2) (1 - f_i)} + f_i + \frac{1}{2} \frac{l_2 + (1 - l_1 - l_2) e_i^2}{l_2 + (1 - l_1 - l_2) e_i} \right) \quad (16)$$

and, similarly, for $e_i < f_i$;

(ii) *if Incumbent moves first, he chooses E if and only if $e_i > f_i$, and then Challenger chooses the same issue as Incumbent;*

(iii) *if Incumbent moves second, then Challenger chooses E if and only if $e_c > f_c$, and Incumbent chooses the same issue as Challenger.*

Moreover, for any $l_2 > 0$, if $l_1 \rightarrow 0$ then the share of Incumbents who opt to move second conditional on having a choice converges to 1, and for any $l_1 > 0$, if $l_2 \rightarrow 0$ then the share of Incumbents who opt to move second conditional on having a choice remains bounded away from zero.

Incumbent's strategies from the equilibrium described in Proposition 14 are depicted on Figure 9 (Right). The restriction $l_1 + l_2 \leq \frac{1}{10}$ guarantees that the two regions where Incumbent starts campaign immediately have non-overlapping projections on the two axes, which simplifies Bayesian updating considerably, and the condition $\mu \leq \frac{1}{6}$ guarantees that it is equilibrium for Incumbents to choose the same issue as Challenger, even though the distribution of their types by that time is non-uniform, as some types would start campaign immediately if given this opportunity. Under

these conditions, formula (16) applies and is easy to understand. For example, if type (e_i, f_i) is indifferent between moving first and second, then moving first will tell voters his true competence on E , e_i ; conditional on this information, there is $\frac{l_1 e_i}{l_1 e_i + (1 - l_1 - l_2) f_i}$ chance that he had to move first (only types with foreign policy competence less than e_i campaign on E if they move have to move first), and $\frac{(1 - l_1 - l_2) f_i}{l_1 e_i + (1 - l_1 - l_2) f_i}$ chance that he had discretion (only types with foreign policy competence less than f_i start with campaigning on E if they have discretion). In the first case, his average competence on foreign policy is $\frac{e_i}{2}$, in the second case, it is $\frac{f_i}{2}$. Now, taking weighted average produces the left-hand side of (16), and its right-hand side follows from similar considerations.

The last results, on limits as $l_1 \rightarrow 0$ or $l_2 \rightarrow 0$, are particularly interesting. If l_1 is very close to 0, then very few Incumbents have to move first, and moving first is very likely to be attributed to a deliberate choice. In that case, Incumbent's competence on the issue he is not campaigning on will be very heavily discounted, and very few incumbents will choose to start an immediate campaign nonetheless. In the limit, almost all Incumbents prefer to wait. In other words, while making a first announcement is an attractive option to at least some politicians, not using this option is even more attractive (unless they have an excuse to use it), because it serves as a positive signal of possessing balanced competence in the two issues. This insight may be extended further, to a game where both politicians get, e.g., alternating opportunities to move first; backward induction will immediately suggest that both politicians will wait until the very last opportunity, and on the equilibrium path, waiting will be something expected rather than a positive signal. (Interestingly, once one politician starts the campaign, the other one has no incentive to wait further.) On the other hand, if l_2 is close to 0, the shares of Incumbents who move first or second are both bounded away from zero: the fact that moving second will be attributed to well-roundedness rather than to having to move second increases attractiveness of waiting, but it does not completely eliminate incentives for very asymmetric Incumbents to move first.

The insights obtained in these extreme cases suggest the following implications. First, politicians are not likely to seize the very first opportunity to pick an issue, and the reason is not aggregate uncertainty (i.e., they might want to learn, which issues voters find most important), but rather signaling considerations. Second, politicians will use opportunities which present a good reason not to wait. For example, if some event or story makes it impossible or very hard for a politician not to react, the politician might well make the first move (e.g., a stock market crash creates a good reason to start campaigning on economy). A politician may also want to use an opportunity which would otherwise go unnoticed (and therefore he would not get enough credit for waiting). If none of these events gets realized, politicians are likely to wait until the last moment, when the remaining time becomes a binding constraint.²⁸

²⁸Starting early may have a tangible benefit, for example, politicians who start their campaign early deliver their message to more voters, or get more donations. Building these motives into the study of campaign dynamics seems

6 Conclusion

This paper studies issue selection by politicians if voters are Bayesian. Issue choice cannot change voter preferences, but it can affect the information the voters possess about the politician and thereby influence the outcome of elections. Two assumptions are crucial: first, a politician may only choose one issue (at least at a time); second, voters learn weakly more if both politicians campaign on the same issue than on different issues. Under these assumptions (and a number of technical ones, such as uniformity of the distributions of types), equilibria are fully characterized and a number of results are obtained. In particular, if campaigns on different issues are uninformative, there is unraveling and campaigning on different issues does not happen in equilibrium; however, if such campaigns are relatively informative, they are more likely to occur than campaigns on the same issue. These choices by politicians carry substantial information value: in particular, voters would not be better off if they could require the politicians to campaign on a given fixed issue. If politicians are allowed to switch issues in the course of a campaign, then the most competent and “well-rounded” politicians will do so, thereby signaling well-roundedness and overall competence, whereas those who excel in one issue only will not. The model allows for a number of other comparative statics results and extensions.

The model is very simple, with only two actors making binary choices (plus a continuum of voters participating in standard probabilistic voting), yet it delivers a surprisingly rich set of results. It is also surprisingly tractable, with simple closed-form solutions in many cases, even though there is ex ante uncertainty over four dimensions. The results obtained in the paper do not appear to be knife-edge; this suggests that the results are likely robust to departures from the model’s assumptions—for example, to nonuniform distribution of types or to voters who are not perfectly Bayesian but exhibit some behavioral traits. These properties of the model make multiple extensions possible and allow for modeling political campaigns in a simple way as part of a larger political economy model. For example, how does the anticipation of a political campaign affect the incumbent’s behavior and, in particular, the choice of issues to address while in office? Or, which politicians value campaign contributions more, if money allows them to buy more of voters’ attention and campaign on multiple issues instead of only one? Or, in a dynamic model of campaigns, how do politicians respond to a change in issue salience due to an exogenous shock? These are all natural questions that the model may help address. More broadly, the insights from the model may be useful both for theoretical research, such as that on strategic disclosure or persuasion, and applied work beyond political economy, such as studying advertising campaigns.

to be another interesting direction for future research.

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Appendix A — Main Proofs

Introduce the following notation. For a politician $j \in \{i, c\}$ (Incumbent or Challenger) with type $(e_j, f_j) = (x, y)$, let $\hat{f}_j(x) = \mathbb{E}(f_j \mid d_j = E, e_j = x)$, and let $\hat{e}_j(y) = \mathbb{E}(e_j \mid d_j = F, f_j = y)$. In other words, $\hat{f}_j(x)$ is voters' equilibrium expectation of f_j conditional on j choosing E and revealing the true value of $e_j = x$, and $\hat{e}_j(y)$ is voters' expectation of e_j conditional on j choosing F and revealing $f_j = y$ (for Challenger, these values also may depend on Incumbent's choice d_i that is known to Challenger at the time of decision-making, but we suppress the argument d_i for brevity). Furthermore, denote the voters' expectation of j 's total competence under these scenarios by $\hat{a}_j^E(x) = x + \hat{f}_j(x)$ and $\hat{a}_j^F(y) = \hat{e}_j(y) + y$. Let \tilde{a}_j^E and \tilde{a}_j^F be the voters' expectations of j 's total competence if he chose E and F , respectively, but no further signal was obtained. Let ξ_j^E and ξ_j^F be the probabilities that politician j will be able to communicate his competence if he chooses E and F , respectively; these values satisfy $\xi_E, \xi_F \geq \mu$. Denote the voters' expectation of his competence taking this into account by $\bar{a}_j^E(x) = \xi_j^E \hat{a}_j^E(x) + (1 - \xi_j^E) \tilde{a}_j^E$ and $\bar{a}_j^F(y) = \xi_j^F \hat{a}_j^F(y) + (1 - \xi_j^F) \tilde{a}_j^F$.

Slightly abusing notation, let E_j be the set of (e_j, f_j) such that $d_j(e_j, f_j) = E$ and F be the set of (e_j, f_j) such that $d_j(e_j, f_j) = F$. In other words, (re)define $E_j, F_j \subset \Omega_j$; $E = \{(x, y) \in \Omega_j : d_j(x, y) = E\}$, and $F = \Omega_j \setminus E_j$. Let $G_j = \bar{E}_j \cap \bar{F}_j = \partial E_j \cap \partial F_j$ be the set of points that have points from both E_j and F_j in any ε -neighborhood (we drop subscript j and write G when this does not cause confusion). Finally, define the following function.

$$\Phi_j(x, y) = \xi_j^E \left(x + \frac{y}{2} \right) + (1 - \xi_j^E) \tilde{a}_j^E - \xi_j^F \left(y + \frac{x}{2} \right) - (1 - \xi_j^F) \tilde{a}_j^F \quad (\text{A1})$$

(again, we will typically drop subscript j). Results from Appendix B establish the relations between $\Phi_j(x, y)$ and G_j under different conditions.

Proof of Lemma 1. As shown in the text, Incumbent maximizes $\mathbb{E}(D(\mathcal{I}) \mid e_i, f_i)$ given by (6). The second term, $\mathbb{E}(\mathbb{E}(U(e_c, f_c) \mid \mathcal{I}) \mid e_i, f_i)$, equals the unconditional expectation $\mathbb{E}(U(e_c, f_c)) = 1$ both if $d_i = E$ and if $d_i = F$, i.e., it is a constant. Thus, d_i maximizes $\mathbb{E}(D(\mathcal{I}) \mid e_i, f_i)$ if and only if it maximizes (8). Similarly, Challenger minimizes $\mathbb{E}(D(\mathcal{I}) \mid e_c, f_c; d_i)$ given by (7), taking d_i as given, but the first term satisfies $\mathbb{E}(\mathbb{E}(U(e_i, f_i) \mid \mathcal{I}) \mid e_c, f_c; d_i) = \mathbb{E}(U(e_i, f_i) \mid d_i)$, again by the law of iterated expectations. The latter value depends on d_i but not on d_c , and therefore is a constant from Challenger's perspective. Hence, his problem is equivalent to maximizing (9). ■

Proof of Proposition 1. For politician j , let ξ_j^E and ξ_j^F denote the probabilities of signaling his competence credibly (i.e., that $\kappa_j \neq \emptyset$), as perceived by him at the time he makes decision, if he chooses E and F , respectively.

Suppose that $d_i = E$. For Challenger, we then have $\xi_c^E = 1$ and $\xi_c^F = \mu$; let $r_c = \xi_c^E / \xi_c^F = \frac{1}{\mu} \in [1, 2)$. Now, Lemma B8 in Appendix B implies that both E and F are chosen by a positive

measure of Challengers. Then by Lemma B10, G is straight line with slope $\frac{2r_c-1}{2-r_c} = \frac{2-\mu}{2\mu-1} \geq 1$; moreover, Lemma B6 implies that this line is defined by $\Phi(x, y) = 0$. Now, we have $\xi_c^E = 1$ and $\xi_j^F = \mu$, thus Lemma B9 is applicable and it suggests that for some $(x, y) \in G \setminus \partial\Omega_c$, $x = y$. This, together with the fact that the slope of G is at least 1, implies that the two endpoints must lie on the lower and upper sides of $\partial\Omega_c$: namely, it must be that $(\alpha, 0) \in G$ and $(\beta, 1) \in G$, and thus $\Phi(\alpha, 0) = \Phi(\beta, 1) = 0$, where Φ is given by (A1). Then direct computations show that $\tilde{\alpha}_c^F = \frac{\alpha^2 + \alpha\beta + \beta^2}{3(\alpha + \beta)} + \frac{\alpha + 2\beta}{3(\alpha + \beta)}$. The conditions on Φ may thus be rewritten as

$$\begin{cases} \alpha = \mu \frac{\alpha}{2} + (1 - \mu) \frac{\alpha^2 + \alpha\beta + \alpha + \beta^2 + 2\beta}{3(\alpha + \beta)}, \\ \beta + \frac{1}{2} = \mu \left(1 + \frac{\beta}{2}\right) + (1 - \mu) \frac{\alpha^2 + \alpha\beta + \alpha + \beta^2 + 2\beta}{3(\alpha + \beta)}. \end{cases}$$

For $\mu > \frac{1}{2}$, this system has a unique solution $\alpha = \frac{4-5\mu + \sqrt{\mu(8-7\mu)}}{8-4\mu}$, $\beta = \frac{3\mu + \sqrt{\mu(8-7\mu)}}{8-4\mu}$. This defines Challenger's strategies uniquely, up to the indifferent types that lie on line G . Trivial algebraic manipulation yields (10). Similarly, if $d_i = F$, Challenger will follow symmetric strategies.

To find Incumbent's strategies, notice that the probability that Challenger chooses the same issue is found in the proof of Proposition 2; it equals $\pi = \frac{3(2-\mu) - \sqrt{\mu(8-7\mu)}}{4(2-\mu)}$. Thus, from Incumbent's perspective, $\xi_i^E = \xi_i^F = \pi + (1 - \pi)\mu = \frac{\mu+3}{4} - \frac{(1-\mu)\sqrt{8\mu-7\mu^2}}{4(2-\mu)} > \frac{2}{3}$ for all μ . This implies that for Incumbent $r_i = 1$, and now Lemmas B8, B10, B9 imply that equilibrium strategies are unique, and his G is a line connecting $(0, 0)$ and $(1, 1)$. ■

Proof of Proposition 2. If $\mu > \frac{1}{2}$, the probability that Challenger chooses issue F when Incumbent chose E (or vice versa) equals

$$\frac{1}{2} \left(\frac{4 - 5\mu + \sqrt{\mu(8 - 7\mu)}}{8 - 4\mu} + \frac{3\mu + \sqrt{\mu(8 - 7\mu)}}{8 - 4\mu} \right) = \frac{2 - \mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)};$$

this follows from Proposition 1. Differentiating this with respect to μ yields

$$\frac{d}{d\mu} \left(\frac{2 - \mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)} \right) = \frac{4 - 5\mu}{2(2 - \mu)^2 \sqrt{\mu(8 - 7\mu)}}.$$

This is increasing for $\mu < \frac{4}{5}$ and decreasing for $\mu > \frac{4}{5}$, and the maximal value equals $\frac{1}{4} + \frac{\sqrt{3}}{6} \approx 0.539$. The probability that a politician communicates his competence on the chosen dimension successfully equals $\frac{\mu+3}{4} - \frac{(1-\mu)\sqrt{8\mu-7\mu^2}}{4(2-\mu)}$, which is increasing on $(\frac{1}{2}, 1)$. This completes the proof. ■

Proof of Proposition 3. Suppose $d_i = E$. For Challenger, $\xi_c^E = 1$ and $\xi_c^F = \mu$. Given that $0 < \mu \leq \frac{1}{2}$, Lemma B8 implies (since $\xi_c^E/2 + \xi_c^F \leq 1$) that equilibria of type (i) are possible for all $\mu \leq \frac{1}{2}$, and these are the only equilibria where almost all types choose the same issue, i.e., equilibria where almost all challengers choose F are not possible (since $\xi_c^E + \xi_c^F/2 > 1$). Now suppose that both E_c and F_c have positive measures. Then, since $r_c = \xi_c^E/\xi_c^F \geq 2$, Lemma B6

implies that $G \setminus \partial\Omega_c$ is a vertical line, and then Lemma B7 implies that there is an equilibrium where G connects $(\alpha, 0)$ and $(\alpha, 1)$ if and only if $\alpha \in [\mu, 1 - \mu]$. This proves that all equilibria of type (ii) are possible, and that there are no other equilibria. The case $d_i = F$ is considered similarly.

Now suppose Challenger plays symmetric strategies. Then for Incumbent, $\xi_i^E = \xi_i^F > \mu$, and thus $r_i = \xi_i^E / \xi_i^F = 1$. Since in a symmetric equilibrium, Incumbent must choose E and F with positive probabilities, Lemma B10 implies that G is a straight line with slope 1, and then Lemma B9 implies that $(x, y) \in G$ if and only if $x = y$, so, Incumbent's strategy to choose E if $e_i > f_i$ and F if $e_i < f_i$ are indeed best responses. This completes the proof. ■

Proof of Proposition 4. It should be noted that all Lemmas in Appendix B apply to the game with simultaneous moves immediately or with trivial changes in their proofs.

Suppose first that Incumbent uses the symmetric strategy and chooses E if and only if $e_i > f_i$. For Challenger, in this case, $\xi_c^E = \xi_c^F = \frac{1}{2} + \frac{1}{2}\mu > \frac{3}{4}$. In this case, $r_c = 1$, and Lemma B10 implies that it is an equilibrium for Challenger to choose symmetric strategies. Similarly, if Challenger plays symmetric strategies, it is an equilibrium for Incumbent to do so. This proves that a symmetric equilibrium exists for all μ .

Suppose that there is another equilibrium. First, let us prove that both politicians must choose both issues with positive probabilities. Suppose not, e.g., that Incumbent chooses E for almost all his types. Then for Challenger, $\xi_c^E = 1$ and $\xi_c^F = \mu$. Lemma B8 then implies that Challenger must choose both issues with positive probabilities. Moreover, Proposition 1 implies that Challenger has an essentially unique strategy, and this strategy involves choosing E with probability $x \in [\frac{3}{4} - \frac{1}{6}\sqrt{3}, \frac{1}{2}] \subset (\frac{2}{5}, \frac{1}{2}]$. Then for Incumbent, $\xi_i^E = x + \mu(1 - x)$ and $\xi_i^F = 1 - x + \mu x$. Since $x \leq \frac{1}{2}$, $\xi_i^E \leq \xi_i^F$, and thus $\xi_i^E + \frac{1}{2}\xi_i^F \leq \xi_i^F + \frac{1}{2}\xi_i^E$. We have $\xi_i^E + \frac{1}{2}\xi_i^F - 1 = \frac{1}{2}(x + 2\mu - x\mu - 1) > \frac{1}{2}(\frac{2}{5} + 2\mu - \frac{2}{5}\mu - 1) > \frac{1}{2}(\frac{2}{5} + (\frac{8}{5})\frac{1}{2} - 1) = \frac{1}{10} > 0$. Lemma B8 then implies that Incumbents must choose both issues with positive probabilities, which is a contradiction.

Second, let us prove that a politician's separation line cannot be horizontal or vertical. Suppose, to obtain a contradiction, that, e.g., Incumbent chooses E if $e_i > \alpha$ and F if $e_i < \alpha$ (the other case is similar). Now suppose that Challenger chooses E with probability $x \in (0, 1)$. Then, as before, $\xi_i^E = x + \mu(1 - x)$ and $\xi_i^F = 1 - x + \mu x$. For Incumbent's strategy to be equilibrium, we must have $\xi_i^E / \xi_i^F \geq 2$, by Lemma B10. We have $\xi_i^E / \xi_i^F - 2 = \frac{x + \mu(1 - x)}{1 - x + \mu x} - 2 = \frac{3x(1 - \mu) + \mu - 2}{1 - x + \mu x}$; for this to be nonnegative, it must be that $x \geq \frac{2 - \mu}{3(1 - \mu)}$. The right-hand side, however, is a strictly increasing function of μ on $(\frac{1}{2}, 1)$, and its value at $\frac{1}{2}$ is 1. Thus, for $\mu > \frac{1}{2}$, $x \geq \frac{2 - \mu}{3(1 - \mu)}$ is impossible. This contradiction shows that separation lines cannot be horizontal or vertical.

Finally, suppose that both politicians have upward-sloping separation lines on both issues. Suppose that Challenger chooses E with probability x and Incumbent chooses F with probability

y . Without loss of generality, suppose that $|x - \frac{1}{2}| \leq |y - \frac{1}{2}|$ (i.e., Challenger plays a weakly more symmetric strategy), and also that $x \in (\frac{1}{2}, 1]$ (the case $x = \frac{1}{2}$ leads to a symmetric equilibrium, as was shown earlier). For Incumbent then $\xi_i^E = x + \mu(1-x)$ and $\xi_i^F = 1 - x + \mu x$. Let $r_i = \xi_i^E / \xi_i^F = \frac{x + \mu(1-x)}{1-x + \mu x}$; then by Lemma B10, the line G that separates E_i and F_i has slope $\frac{2r_i-1}{2-r_i} = \frac{3x(1-\mu)+2\mu-1}{2-\mu-3x(1-\mu)} \geq 1$ (since $x \geq \frac{1}{2}$). Moreover, since $\xi_i^E + \xi_i^F = x + \mu(1-x) + 1 - x + \mu x = 1 + \mu > \frac{3}{2}$, it must be that $\max(\xi_i^E, \xi_i^F) \geq \frac{2}{3}$, and then Lemma B9 implies that for Incumbent, the separation line G has slope $\frac{2r_i-1}{2-r_i}$ and intersects the main diagonal. Suppose that G intersects with $\partial\Omega_i$ at points $(\alpha, 0)$ and $(\beta, 1)$, then the fact that F_i has measure y implies $\alpha = y - \frac{1}{2} \frac{2-\mu-3x(1-\mu)}{3x(1-\mu)+2\mu-1}$, $\beta = y + \frac{1}{2} \frac{2-\mu-3x(1-\mu)}{3x(1-\mu)+2\mu-1}$. The conditions $\alpha \geq 0$ and $\beta \leq 1$ imply that y must satisfy

$$\frac{1}{2} \frac{2-\mu-3x(1-\mu)}{3x(1-\mu)+2\mu-1} \leq y \leq 1 - \frac{1}{2} \frac{2-\mu-3x(1-\mu)}{3x(1-\mu)+2\mu-1}. \quad (\text{A2})$$

Direct calculations (as in the proof of Proposition 1) imply that $\tilde{a}_i^F = \frac{\alpha^2 + \alpha\beta + \beta^2}{3(\alpha+\beta)} + \frac{\alpha+2\beta}{3(\alpha+\beta)}$, and $\tilde{a}_i^E = \frac{1-\tilde{a}_i^F y}{1-y}$. Now the indifference condition for type $(\alpha, 0)$, $\Phi(\alpha, 0) = 0$, may be written as

$$\xi_i^E \alpha + (1 - \xi_i^E) \tilde{a}_i^E - \xi_i^F \frac{\alpha}{2} - (1 - \xi_i^F) \tilde{a}_i^F = 0.$$

Plugging in $\xi_i^E, \xi_i^F, \tilde{a}_i^E, \tilde{a}_i^F, \alpha, \beta$, and making substitutions $x = u + \frac{1}{2}$, $y = v + \frac{1}{2}$, and $m = 1 - \mu$ yields the following equation on v :

$$v^3 + \frac{m(1-2u)}{2-m(1-2u)} v^2 - \frac{12m^2 u^2 + 16m(2-m)u + (2-m)^2}{4(2-m(1-6u))^2} v + \frac{um(36m^2 u^2 + 12m(2-3m)u + (2-9m)(2-m))}{2(2-m(1-2u))(2-m(1-6u))^2} = 0,$$

where v must satisfy the condition $|v| \leq \frac{6um}{2-m+6um}$. Let us prove that for $m, u \in (0, \frac{1}{2})$, this equation on v has no solutions on $X = \left[-\frac{6um}{2-m+6um}, \frac{6um}{2-m+6um}\right] \setminus [-a, a]$. Denote the right-hand side $f(v)$; we then have

$$f\left(-\frac{6um}{2-m+6um}\right) = 2mu(2-m-6mu) \frac{60m^2 u^2 + 12m(4-3m)u + (2-3m)(2-m)}{(2-m(1-2u))(2-m(1-6u))^3} > 0.$$

If $-\frac{6um}{2-m+6um} \leq -u$, i.e., if $m \leq \frac{2}{7-6u}$, we have

$$f(-u) = \frac{u}{4(2-m+2mu)(2-m+6mu)^2} \left[\begin{array}{l} (8-32u^2) - (224u^3 - 16u^2 - 88u + 4)m - \\ (480u^4 - 32u^3 - 192u^2 + 40u + 34)m^2 + \\ (17-50u-24u^2+208u^3-48u^4-288u^5)m^3 \end{array} \right] > 0;$$

to see this, notice that for a fixed u , the expression in brackets is a cubic with a positive coefficient of m^3 ; the numerator is positive for $m = 0$ and $m = \frac{2}{7-6u}$ (equal to $8(1-2u)(1+2u)$ and $1536(1-2u) \frac{1+3u-2u^2}{(7-6u)^3}$, respectively), and $m = \frac{2}{7-6u}$ is located on the downward-sloping part of the

cubic, as the derivative with respect to m equals $16 \frac{288u^4 - 648u^3 - 44u^2 + 234u - 59}{(7-6u)^2} < 0$; this shows that $f\left(-\frac{6um}{2-m+6um}\right) > 0$ whenever $-\frac{6um}{2-m+6um} \leq v \leq -u$. We similarly have

$$f(u) = \frac{-u(1-4u^2)}{4(2-m+2mu)(2-m+6mu)^2} \left[12m^2(10-11m)u^2 + (2-m)(4-8m+19m^2) \right] < 0;$$

$$f\left(\frac{6um}{2-m+6um}\right) = -mu(2-m-6mu) \frac{12m^2u^2 + 48mu + (2-m)(2+3m)}{(2-m(1-2u))(2-m(1-6u))^3} < 0.$$

Since in $f(\cdot)$, the coefficient of v^3 is positive, this leaves the only possibility, where $f(v) = 0$ has three real roots: one on $\left(-\infty, -\frac{6um}{2-m+6um}\right)$, another on $\left(\frac{6um}{2-m+6um}, +\infty\right)$, and a third one on $\left(\max\left(-u, \frac{6um}{2-m+6um}\right), u\right)$. None of these roots is in set X . However, this implies that the equilibrium v corresponds to the third root, and satisfies $-u < v < u$. However, this implies that $|u| \leq |v|$ is impossible in equilibrium, which contradicts the hypothesis that $|x - \frac{1}{2}| \leq |y - \frac{1}{2}|$. This contradiction implies that there are no other equilibria where both politicians use upward-sloping separation lines, and this completes the proof. ■

Proof of Lemma 2. The expected competence of the elected politician is $\mathbb{E}(\mathbb{E}(U(e_i, f_i) | \mathcal{I})) = \mathbb{E}(U(e_i, f_i)) = 1$ for Incumbent, and similarly for Challenger.

Denote $\Omega = \Omega_i \times \Omega_c$, Let us show that $\int_{\Omega} \mathbb{E}(U(e_i, f_i) | \mathcal{I}) \mathbb{E}(U(e_c, f_c) | \mathcal{I}) d\lambda = 1$. We split the entire space of types into four regions, $(d_i, d_c) \in \{(E, E), (E, F), (F, E), (F, F)\}$ according to the issue chosen in equilibrium (note that these need not be independent, as Challenger's choice depends on that of Incumbent if the moves are sequential). Let $\pi(d_i, d_c)$ be the probability of Incumbent and Challenger choosing d_i and d_c , respectively, and $\pi(d_i) = \pi(d_i, E) + \pi(d_i, F)$. We have

$$\begin{aligned} & \int_{\Omega} \mathbb{E}(U(e_i, f_i) | \mathcal{I}) \times \mathbb{E}(U(e_c, f_c) | \mathcal{I}) d\lambda \\ &= \mathbb{E}[\mathbb{E}(U(e_i, f_i) | \mathcal{I}) \times \mathbb{E}(U(e_c, f_c) | \mathcal{I})] \\ &= \sum_{(d_i, d_c) \in S^2} \pi(d_i, d_c) \mathbb{E}[\mathbb{E}(U(e_i, f_i) | \mathcal{I}) \times \mathbb{E}(U(e_c, f_c) | \mathcal{I}) | d_i, d_c] \\ &= \sum_{(d_i, d_c) \in S^2} \pi(d_i, d_c) \mathbb{E}[\mathbb{E}(U(e_i, f_i) | \mathcal{I}) | d_i, d_c] \times \mathbb{E}[\mathbb{E}(U(e_c, f_c) | \mathcal{I}) | d_i, d_c] \\ &= \sum_{(d_i, d_c) \in S^2} \pi(d_i, d_c) \mathbb{E}[U(e_i, f_i) | d_i, d_c] \times \mathbb{E}[U(e_c, f_c) | d_i, d_c] \\ &= \sum_{d_i \in S} \sum_{d_c \in S} \pi(d_i, d_c) \mathbb{E}[U(e_i, f_i) | d_i] \times \mathbb{E}[U(e_c, f_c) | \Omega_{(d_i, d_c)}] \\ &= \sum_{d_i \in S} \mathbb{E}[U(e_i, f_i) | d_i] \sum_{d_c \in S} \pi(d_i, d_c) \mathbb{E}[U(e_c, f_c) | d_i, d_c] \end{aligned}$$

$$\begin{aligned}
&= \sum_{d_i \in \mathcal{S}} \mathbb{E}[U(e_i, f_i) \mid d_i] \times \pi(d_i) \mathbb{E}[U(e_c, f_c) \mid d_i] \\
&= \sum_{d_i \in \mathcal{S}} \lambda_{d_i} \mathbb{E}[U(e_i, f_i) \mid d_i] \\
&= \mathbb{E}[U(e_i, f_i)] = 1.
\end{aligned}$$

Here, the third equality uses the fact that conditional on the choices d_i and d_c , the expected competences of the two politicians taken from the voters' perspective are independent. The fourth equality uses the law of iterated expectations. The fifth one uses the fact that the expected competence of Incumbent who chose d_i does not depend on the choice of Challenger, because the latter is made either simultaneously or later. The seventh inequality again uses the law of iterated expectations. The eighth one uses the fact that the expected competence of Challenger equals 1 (and does not depend on d_i), and the ninth one again uses the law of iterated expectations. We thus have

$$\begin{aligned}
&1 + A \int_{\Omega} [\mathbb{E}(U(e_i, f_i) \mid \mathcal{I}) - \mathbb{E}(U(e_c, f_c) \mid \mathcal{I})]^2 d\lambda \\
&= 1 + A \left(\int_{\Omega} ([\mathbb{E}(U(e_i, f_i) \mid \mathcal{I})]^2 + [\mathbb{E}(U(e_c, f_c) \mid \mathcal{I})]^2) d\lambda - 2 \right).
\end{aligned}$$

Taking the weighted sum over the two possible realizations of χ , we get (12). This completes the proof. ■

Proof of Proposition 5. Suppose that $\mu > \frac{1}{2}$. We first calculate the integral for Incumbent (again, assuming that $e_i > f_i$ as the opposite case may be considered similarly). With probability $1 - \frac{2-\mu+\sqrt{\mu(8-7\mu)}}{4(2-\mu)}$, Challenger also chooses E . This gives $\int_0^1 (\frac{3}{2}x)^2 2xdx = \frac{9}{8}$. With probability $\frac{2-\mu+\sqrt{\mu(8-7\mu)}}{4(2-\mu)}$, Challenger chooses F , and the integral equals $\mu * \frac{9}{8} + (1-\mu) * 1$. In total, the contribution of Incumbent is

$$\begin{aligned}
\frac{9}{8} \left(1 - \frac{2-\mu+\sqrt{\mu(8-7\mu)}}{4(2-\mu)} \right) + \frac{2-\mu+\sqrt{\mu(8-7\mu)}}{4(2-\mu)} \left(\mu * \frac{9}{8} + (1-\mu) * 1 \right) \\
= \left(\frac{9}{8} - \frac{1-\mu}{32(2-\mu)} (2-\mu+\sqrt{\mu(8-7\mu)}) \right).
\end{aligned}$$

Now, consider the contribution of Challenger. The part of the integral coming from the set where he chooses E is (as before, $\alpha = \frac{4-5\mu+\sqrt{\mu(8-7\mu)}}{8-4\mu}$, $\beta = \frac{3\mu+\sqrt{\mu(8-7\mu)}}{8-4\mu}$):

$$\begin{aligned}
&\int_{\alpha}^{\beta} \left(x + \frac{1}{2} \frac{x-\alpha}{\beta-\alpha} \right)^2 \frac{x-\alpha}{\beta-\alpha} dx + \int_{\beta}^1 \left(x + \frac{1}{2} \right)^2 dx \\
&= \frac{13}{12} - \frac{3\alpha + 9\beta + 4\alpha\beta^2 + 4\alpha^2\beta + 8\alpha\beta + 4\alpha^2 + 4\alpha^3 + 12\beta^2 + 4\beta^3}{48}.
\end{aligned}$$

The part of the integral coming from the set where he chooses F may be found as follows: with probability μ , it is

$$\begin{aligned} & \int_0^1 \left(y + \frac{1}{2} (\alpha + (\beta - \alpha) y) \right)^2 (\alpha + (\beta - \alpha) y) dy \\ &= \frac{1}{48} (4\alpha + 12\beta + 3\alpha\beta^2 + 3\alpha^2\beta + 8\alpha\beta + 4\alpha^2 + 3\alpha^3 + 12\beta^2 + 3\beta^3), \end{aligned}$$

and with probability $1 - \mu$, it is

$$\left(\frac{\alpha^2 + \alpha\beta + \beta^2}{3(\alpha + \beta)} + \frac{\alpha + 2\beta}{3(\alpha + \beta)} \right)^2 \frac{\alpha + \beta}{2} = \frac{1}{18(\alpha + \beta)} (\alpha^2 + \alpha\beta + \alpha + \beta^2 + 2\beta)^2.$$

This means that the total integral for Incumbent and Challenger, after substituting for the values of α and β , is

$$2 + \frac{(68 - 19\mu - 6\mu^2)(2 - \mu)^2 - (24 - 52\mu + 29\mu^2 - 6\mu^3) \sqrt{\mu(8 - 7\mu)}}{192(2 - \mu)^3}.$$

It may be shown directly that this is an increasing function of μ , and that its value for $\mu = \frac{1}{2}$ is $2 + \frac{3}{16}$ and for $\mu = 1$ is $2 + \frac{1}{4}$. The result for $\mu > \frac{1}{2}$ follows.

Consider now the case $\mu \leq \frac{1}{2}$. Let us show that equilibria of type (ii) result in a lower welfare than equilibrium of type (i). Without loss of generality, suppose Incumbent chooses E , and Challenger chooses E if $e_c > \alpha$ and F if $e_c < \alpha$ ($\alpha = 0$ corresponds to type (i) equilibrium and $\alpha \in [\mu, 1 - \mu]$ corresponds to type (ii) equilibrium). The contribution of Incumbent to welfare is given by

$$(1 - \alpha + \alpha\mu) \left(2 \int_0^1 \int_0^x \left(x + \frac{x}{2} \right)^2 dy dx \right) + \alpha(1 - \mu) \left(2 \int_0^1 \int_0^x 1 dy dx \right) = \frac{9 - \alpha + \alpha\mu}{8}.$$

The contribution of Challenger equals

$$\int_\alpha^1 \int_0^1 \left(x + \frac{1}{2} \right)^2 dy dx + \mu \int_0^\alpha \int_0^1 \left(y + \frac{\alpha}{2} \right)^2 dy dx + (1 - \mu) \int_0^\alpha \int_0^1 \left(\frac{1}{2} + \frac{\alpha}{2} \right)^2 dy dx = \frac{13 + \alpha\mu - \alpha^3}{12}.$$

The sum of the two equals

$$\frac{53 - 3\alpha + 5\alpha\mu - 2\alpha^3}{24},$$

which is a decreasing function of α (its derivative with respect to α is $-\frac{3-5\mu+6\alpha^2}{24} < 0$ for $\mu \leq \frac{1}{2}$).

This means that the maximum is reached at $\alpha = 0$, i.e., in equilibrium of type (i), and this maximum equals $\frac{53}{24} = 2 + \frac{5}{24}$. Plugging this value of the integral into (12) yields the result for this case. This completes the proof. ■

Proof of Proposition 6. In the symmetric equilibrium, each politician is able to announce his competence credibly with probability $\frac{1}{2} + \frac{1}{2}\mu$, and he fails to do so with probability $\frac{1}{2}(1 - \mu)$. Consequently, the integral in the right-hand side of (12) equals

$$2 \left(\left(\frac{1}{2} + \frac{1}{2}\mu \right) \int_0^1 \left(\frac{3}{2}x \right)^2 2x dx + \frac{1}{2}(1 - \mu) \int_0^1 2x dx \right) = 2 + \frac{1 + \mu}{8}.$$

Therefore, the expected competence of the elected politician equals $1 + \frac{1+\mu}{8}A$, and is thus monotonically increasing in μ . Its maximum is achieved at $\mu = \frac{1}{2}$ and it equals $1 + \frac{3}{16}A$, which exceeds $1 + \frac{5}{24}A$, the expected competence in the sequential game (see Proposition 5). If $\mu > \frac{1}{2}$, then the proof of Proposition 5 suggests that the expected competence of the elected politician in the sequential game is

$$1 + A \frac{(68 - 19\mu - 6\mu^2)(2 - \mu)^2 - (24 - 52\mu + 29\mu^2 - 6\mu^3)\sqrt{\mu(8 - 7\mu)}}{192(2 - \mu)^3},$$

which is less than $1 + \frac{1+\mu}{8}A$ for $\mu \in (\frac{1}{2}, 1)$, as the difference equals

$$A \frac{(20 - 43\mu + 18\mu^2)(2 - \mu)^2 - (24 - 52\mu + 29\mu^2 - 6\mu^3)\sqrt{\mu(8 - 7\mu)}}{192(2 - \mu)^3},$$

which is negative for such μ . This completes the proof. ■

Proof of Proposition 7. If an issue is fixed exogenously, the expected competence of the elected politician is given by (14) and equals $1 + \frac{1}{6}A$. This is less than $1 + \frac{3}{16}A$, which is the lower bound of utilities of both the sequential and simultaneous games for $\mu > \frac{1}{2}$. On the other hand, if $\mu \leq \frac{1}{2}$, then the payoff in the type (i) equilibrium of the sequential game is $1 + \frac{5}{24}A$, which is even higher. However, the payoff in the case of the simultaneous game is $1 + \frac{1+\mu}{8}A$, and it is lower than $1 + \frac{1}{6}A$ if and only if $\mu < \frac{1}{3}$. This completes the proof. ■

Proof of Proposition 8. The expected probability of challenger winning is obtained by taking the expectation of (5) over all possible realizations of (e_c, f_c) , (e_i, f_i) , as well as κ_c and κ_i . The law of iterated expectations implies that this equals $\frac{1}{2}$, and thus the expected probability of incumbent winning also equals $\frac{1}{2}$. This completes the proof. ■

Proof of Proposition 9. In the symmetric equilibrium, Incumbent chooses $\tilde{d}_i = E$ if $e_i > f_i$ and $\tilde{d}_i = F$ if $e_i < f_i$. Consider the case where $\tilde{d}_i = E$ and $d_c = F$. If Incumbent chooses $d_i = F$, he conveys his competence with probability $\xi_i^{EF} = 1$; if he chooses E , he does so with probability $\xi_i^{EE} = \mu$. Similarly to the original Challenger's problem (Lemmas B4 and B10), one can show that the regions EE_i where Incumbent sticks to E and EF_i where he switches to F are separated by an upward-sloping straight line (with types that switch lying above and left of the types that do not), and types on this line are indifferent. Consider an agent with competence (x, y) and suppose that he is indifferent. If so, then if he chooses E , he gets $\mu(x + \frac{y}{2}) + (1 - \mu)\tilde{a}_i^{EE}$, where $\tilde{a}_i^{EE} = \mathbb{E}(e_i + f_i \mid \tilde{d}_i(e_i, f_i) = d_i(e_i, f_i) = E, d_j = F)$. If he chooses F instead, he gets $y + \frac{x+y}{2}$. Consequently, he is indifferent if and only if

$$y = \frac{2\mu - 1}{3 - \mu}x + \frac{2(1 - \mu)}{3 - \mu}\tilde{a}_i^{EE}. \quad (\text{A3})$$

This indeed defines an upward-sloping line for $\frac{1}{2} < \mu \leq 1$. Moreover, for such μ , $x > 0$ implies $y > 0$. Consequently, if the measures of Incumbents who stay and who switch are both positive (the other cases are easy to rule out similarly to Lemma B8), the line should connect some point (α, α) with some point $(1, \beta)$. Moreover, it must be that $\beta = \alpha + \frac{2\mu-1}{3-\mu}(1-\alpha)$ and $0 \leq \alpha < \beta < 1$.

Let us now find \tilde{a}_i^{EE} as a function of α . The area EE_i consists of two triangles, obtained from the bottom-right triangle by connecting (α, α) with $(1, 0)$. One triangle has vertices $(0, 0), (1, 0), (\alpha, \alpha)$; its area is $\frac{\alpha}{2}$ and the sum of the coordinates of its mass center is $\frac{1}{3}(1 + \alpha + \alpha) = \frac{2\alpha+1}{3}$. The other triangle has vertices $(1, \beta), (1, 0), (\alpha, \alpha)$; its area is $\frac{(1-\alpha)\beta}{2} = \frac{(1-\alpha)(\alpha + \frac{2\mu-1}{3-\mu}(1-\alpha))}{2}$ and the sum of coordinates of its mass center is $\frac{1}{3}(1 + 1 + \alpha + \beta + \alpha) = \frac{2\alpha+\beta+2}{3} = \frac{2\alpha+(1-\alpha)(\alpha + \frac{2\mu-1}{3-\mu}(1-\alpha))+2}{3}$. Therefore,

$$\tilde{a}_i^{EE} = \frac{1}{3} \frac{-2\alpha^2\beta + 2\alpha^2 - \alpha\beta^2 + \alpha + \beta^2 + 2\beta}{\alpha + \beta - \alpha\beta}.$$

Thus, (A3), when evaluated at point $(x, y) = (\alpha, \alpha)$, simplifies to

$$\begin{aligned} H(\alpha, \mu) &= (3\mu^3 + 23\mu^2 - 84\mu + 64)\alpha^3 + (288\mu - 108\mu^2 - 192)\alpha^2 \\ &+ (81\mu^2 - 171\mu + 84)\alpha + (30\mu - 20\mu^2 - 10) = 0. \end{aligned} \quad (\text{A4})$$

If $\mu = 1$, the equation (A4) has a unique root on $[0, 1)$, $\alpha = 0$, and in this case the indifference line connects $(0, 0)$ and $(1, \frac{1}{2})$. In the other extreme, $\mu = \frac{1}{2}$, we must have $\alpha = \beta$, and thus $\alpha > 0$. In this case, the only root is $\alpha = \frac{4-\sqrt{10}}{3} \approx 0.28$; obviously, this case is the limit as μ tends to $\frac{1}{2}$ as well.

Let us show that for all $\mu \in (0, 1)$, there is a unique root $\alpha \in (0, 1)$. The left-hand side of (A4) equals $10(2\mu - 1)(1 - \mu) > 0$ if $\alpha = 0$ and it equals $-3(2 - \mu)(3 - \mu)^2 < 0$ if $\alpha = 1$. Thus, there exists a root $\alpha \in (0, 1)$ for any such μ . Moreover, if $\mu = 1$, the three roots of the cubic equation are $\{1 - \sqrt{2}, 0, 1 + \sqrt{2}\}$, and so $\alpha = 0$ is a simple root. Therefore, for μ close to 1, the equation (A4) has a unique simple root on $(0, 1)$. If for some $\mu \in (\frac{1}{2}, 1)$ there are two or three roots on $(0, 1)$, then at least one of the following three alternatives must be true: either for some value of $\mu \in (\frac{1}{2}, 1)$, $\alpha = 0$ is a root, or $\alpha = 1$ is a root, or there is a double root $\alpha' \in (0, 1)$. The first two possibilities are ruled out, because $\alpha = 0$ may be a root only for $\mu \in \{\frac{1}{2}, 1\}$, and $\alpha = 1$ may be a root only for $\mu \in \{2, 3\}$. For the last case, suppose that some $\alpha' \in (0, 1)$ is a double root, then the second derivative of (A4) must vanish at some α between the single root and the new double root:

$$6(3\mu^3 + 23\mu^2 - 84\mu + 64)\alpha + 2(288\mu - 108\mu^2 - 192) = 0. \quad (\text{A5})$$

Since $\mu \neq \frac{4}{3}$, this is equivalent to

$$\alpha = \frac{12\mu - 16}{9\mu + \mu^2 - 16}, \quad (\text{A6})$$

which decreases from $\frac{8}{9}$ to $\frac{2}{3}$ as μ increases from $\frac{1}{2}$ to 1. For α given by (A6), (A4) simplifies to

$$\frac{-2(3-\mu)^2}{(16-9\mu-\mu^2)^2} (10\mu^4 - 261\mu^3 + 1053\mu^2 - 1536\mu + 768) = 0.$$

It is straightforward to check that the last factor has no roots on $(\frac{1}{2}, 1)$; this proves that the root is unique.

We can show that this root is decreasing in μ . Indeed, the root $\alpha = 0$ for $\mu = 1$ satisfied $\frac{\partial H(\alpha, \mu)}{\partial \alpha} < 0$; since we proved that there is no double root, we must have that $\frac{\partial H(\alpha, \mu)}{\partial \alpha} < 0$ for all μ . It remains to show that $\frac{\partial H(\alpha, \mu)}{\partial \mu} < 0$ at any root. We have

$$\frac{\partial H(\alpha, \mu)}{\partial \mu} = (9\mu^2 + 46\mu - 84)\alpha^3 + (288 - 216\mu)\alpha^2 + (162\mu - 171),$$

Let us show that it is positive for all $\mu \in [\frac{1}{2}, 1]$, $\alpha \in [0, \frac{4-\sqrt{10}}{3}]$. Indeed, for such values, $\frac{\partial H(\alpha, \mu)}{\partial \mu}$ is increasing in μ :

$$\begin{aligned} \frac{\partial^2 H(\alpha, \mu)}{\partial \mu^2} &= 9\alpha^3\mu - 108\alpha^2 + 23\alpha^3 + 81 \\ &\geq \frac{9}{2}\alpha^3 - 108\alpha^2 + 23\alpha^3 + 81 \\ &\geq 27\alpha^3 - 108\alpha^2 + 81 = 27(1-\alpha)(3+3\alpha-\alpha^2) > 0. \end{aligned}$$

Consequently, it remains to prove that

$$\left. \frac{\partial H(\alpha, \mu)}{\partial \mu} \right|_{\mu=1} = 72\alpha^2 - 29\alpha^3 - 9 < 0$$

for $\alpha \in [0, \frac{4-\sqrt{10}}{3}]$, which is straightforward.

We can also prove that β is increasing in μ . Indeed, given $\alpha = \frac{3\beta-2\mu-\beta\mu+1}{4-3\mu}$, we can rewrite (A4) as

$$\begin{aligned} \tilde{H}(\beta, \mu) &= (\mu^3 + 6\mu^2 - 43\mu + 48)\beta^3 + (6\mu^3 - 57\mu^2 + 165\mu - 144)\beta^2 \\ &\quad + (12\mu^3 - 57\mu^2 + 63\mu)\beta + (8\mu^3 - 18\mu^2 + 7\mu) = 0. \end{aligned} \tag{A7}$$

This function has no double root: otherwise there would be a point with $\frac{\partial^2 \tilde{H}(\beta, \mu)}{\partial \beta^2} = 0$, in which case $\beta = \frac{16-13\mu+2\mu^2}{16-9\mu-\mu^2}$; plugging this into (A7) yields

$$\frac{-2(4-3\mu)^2}{(16-9\mu-\mu^2)^2} (10\mu^4 - 261\mu^3 + 1053\mu^2 - 1536\mu + 768),$$

which, as we know, has no roots on $\mu \in (\frac{1}{2}, 1)$. Now, this means that $\frac{\partial \tilde{H}(\beta, \mu)}{\partial \beta} < 0$ for all μ , because this is true for $\mu = 1$. It remains to prove that $\frac{\partial \tilde{H}(\beta, \mu)}{\partial \mu} > 0$. We have

$$\frac{\partial \tilde{H}(\beta, \mu)}{\partial \mu} = (3\mu^2 + 12\mu - 43)\beta^3 + (18\mu^2 - 114\mu + 165)\beta^2 + (36\mu^2 - 114\mu + 63)\beta + (24\mu^2 - 36\mu + 7).$$

One can show that this is positive at the root β .

We have shown that the subgame where Incumbent gets a chance to reconsider his original choice has a unique equilibrium, so Incumbents who originally chose E and F act symmetrically. Let η be the probability that Incumbent switches conditional on being given such a chance. Consider now the choice of Challenger; for concreteness, suppose that Incumbent chose E . If he chooses $d_c = E$, then his chance of showing his competence is $\xi_c^E = 1$, whereas if he chooses $d_c = F$, then the corresponding probability is $\xi_c^F = p\eta + ((1-p) + p(1-\eta))\mu = \mu + p\eta(1-\mu) > \frac{1}{2}$ (notice that the original game corresponds to the case where Incumbent never reconsiders or never gets such a chance, i.e., $p\eta = 0$ and $\xi_c^F = \mu$). This means that his ratio $r_c = 1/\mu' \in (1, 2)$, and thus Lemma B10 is applicable, leading to the same characterization as in Proposition 1 after replacing μ with μ' . In particular, this means that Challenger chooses the same issue as Incumbent with probability at least $\pi \geq \frac{3}{4} - \frac{\sqrt{3}}{6}$, as follows from Proposition 2.

Finally, consider Incumbent's original problem. Since for $p < 4\sqrt{3} - 6$ we have $p + (1-p)\pi > \frac{1}{2}$, which means that regardless of the strategy this type of Incumbent uses in the following round, the probability that he will eventually campaign on E is higher if he chooses E . This already implies that symmetric strategies form an equilibrium. This completes the proof. ■

Proof of Proposition 10. Without loss of generality, assume that $e_2 - e_1 \leq f_2 - f_1$ (the opposite case is symmetric). If so, renormalize the rectangle $[e_1, e_2] \times [f_1, f_2]$ by mapping $(x, y) \mapsto \left(\frac{x-e_1}{f_2-f_1}, \frac{y-f_1}{f_2-f_1}\right)$; we then have a uniform distribution on $[0, m] \times [0, 1]$, where m is the ratio $\frac{e_2-e_1}{f_2-f_1}$. Since this is an affine transformation of Incumbent's competence, it does not affect his or Challenger's maximization problem.

If Incumbent chooses E (correspondingly, F), Challenger will campaign on E (correspondingly, F) with probability $\pi = \frac{3(2-\mu) - \sqrt{\mu(8-7\mu)}}{4(2-\mu)}$, and will thus be able to reveal his competence credibly with probability $\xi_i^E = \xi_i^F = \pi + (1-\pi)\mu > \frac{2}{3}$. Then the ratio $r_i = \xi_i^E/\xi_i^F = 1$. Consider first equilibria where both E and F are picked by Incumbent with positive probability. In this case, as in Lemma B6, we can prove that the line separating E_i and F_i must be upward-sloping with slope 1, and the points on this line must satisfy $\Phi(x, y) = \Phi_i(x, y) = 0$.

Suppose, to obtain a contradiction, that $\Phi(0, 0) > 0$, so that the line $\Phi(x, y) = 0$ lies above the line $y = x$. Then, evidently, $\tilde{a}_i^F > \tilde{a}_i^E$, but if so, we would have $\Phi(0, 0) = (1 - \xi_i^E)(\tilde{a}_i^E - \tilde{a}_i^F) < 0$, a contradiction. Thus, $\Phi(0, 0) \leq 0$, and the intersection of the line $\Phi(x, y) = 0$ with $\partial\tilde{\Omega}_i$ are some points $(\alpha, 0)$ and $(m, m - \alpha)$, with $0 \leq \alpha < m$. Denoting $\xi = \xi_i^E = \xi_i^F$, we get the following condition for $\Phi(\alpha, 0) = 0$:

$$\xi\alpha + (1 - \xi)\tilde{a}_i^E = \xi\left(\frac{\alpha}{2}\right) + (1 - \xi)\tilde{a}_i^F.$$

This is equivalent to

$$\frac{1-\xi}{\xi} (\tilde{a}_i^F - \tilde{a}_i^E) = \frac{\alpha}{2}.$$

We have $\tilde{a}_i^E = \alpha + \frac{m-\alpha}{3} + \frac{2}{3}(m-\alpha) = m$, and \tilde{a}_i^F may be found from

$$\tilde{a}_i^E \frac{(m-\alpha)^2}{2m} + \tilde{a}_i^F \left(1 - \frac{(m-\alpha)^2}{2m}\right) = \frac{m+1}{2}.$$

This implies that $\tilde{a}_i^F = m \frac{1+2m-(m-\alpha)^2}{2m-(m-\alpha)^2}$, and thus $\tilde{a}_i^F - \tilde{a}_i^E = \frac{m(1-m)}{2m-(m-\alpha)^2}$. Consequently, α is found from the equation

$$\frac{1-\xi}{\xi} \frac{m(1-m)}{2m-(m-\alpha)^2} = \frac{\alpha}{2}; \quad (\text{A8})$$

since the left-hand side is decreasing in α , it has at most one solution. On the other hand, there is a solution $\alpha \in [0, m)$ if and only if $\xi + m > 1$, i.e., $m > 1 - \xi$. Moreover, if $m = 1$ (i.e., if $e_2 - e_1 = f_2 - f_1$ and Ω'_i is a square), then $\alpha = 0$, and if $m < 1$, then $\alpha > 0$.

Consider the possibility of equilibria where all or almost all types choose the same issue. If all types choose F , then $\tilde{a}_i^F = \frac{m+1}{2}$, and the condition that type $(m, 0)$ does not deviate is

$$\xi \left(m + \tilde{f}_i(m)\right) + (1-\xi) \tilde{a}_i^E \leq \xi \left(\frac{m}{2}\right) + (1-\xi) \frac{m+1}{2}.$$

This is satisfied (for $\tilde{f}_i(m) = \tilde{a}_i^E = 0$) if $m \leq \frac{1-\xi}{2\xi-1}$. If all types choose E , then $\tilde{a}_i^E = \frac{m+1}{2}$, and the condition that type $(0, 1)$ does not deviate is

$$\xi \left(\frac{1}{2}\right) + (1-\xi) \frac{m+1}{2} \geq \xi (1 + \tilde{e}_i(1)) + (1-\xi) \tilde{a}_i^F.$$

Since $\xi > \frac{2}{3}$, this is satisfied (for $z_1 = \tilde{a}_i^F = 0$) only if $m \geq \frac{2\xi-1}{1-\xi} > 1$, which is impossible. Thus, only equilibria where issue F is chosen by almost all Incumbents are possible, and this is the case for $m \leq \frac{1-\xi}{2\xi-1}$.

Therefore, equilibria have the following structure: there is a unique equilibrium with both E and F chosen with a positive probability if $m > \frac{1-\xi}{2\xi-1}$, a unique equilibrium where almost all incumbents choose F if $m < 1 - \xi$, and both these equilibria if $1 - \xi < m \leq \frac{1-\xi}{2\xi-1}$. In addition, $m = 1$ implies $\alpha = 0$, which proves (i), and $m < 1$ implies $\alpha > 0$, which, coupled with the characterization of equilibria, proves (ii). It remains to show that a lower m implies a lower probability of Incumbent campaigning on E . Notice first that a decrease in m can only make equilibrium where (almost) all Incumbents choose F appear, and equilibrium where a positive share of Incumbents choose E disappear. It thus suffices to prove that in the latter type of equilibria, a decrease in m leads to a decrease in the probability that Incumbent chooses E . To show this, it suffices to show that α is a decreasing function of m . To show this, rewrite (A8) as $g(\alpha, m) = 0$, where

$$g(\alpha, m) = \frac{\xi}{1-\xi} m(1-m) - \frac{\alpha}{2} \left(2m - (m-\alpha)^2\right).$$

Differentiating with respect to α and m at points where $g(\alpha, m) = 0$, we have

$$\begin{aligned}\frac{\partial}{\partial \alpha} g(\alpha, m) &= \frac{1}{2} (3\alpha^2 + m^2 - 4m\alpha - 2m) \\ &= -\frac{1}{2} \left(2m - (m - \alpha)^2 + 2\alpha(m - \alpha) \right) \\ &= -\frac{1}{\alpha} \frac{\xi}{1 - \xi} m(1 - m) - \alpha(m - \alpha) < 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial}{\partial m} g(\alpha, m) &= -\left((2m - 1) \frac{\xi}{1 - \xi} + \alpha + \alpha^2 - m\alpha \right) \\ &= -\left((2m - 1) \frac{\alpha}{2} \frac{2m - (m - \alpha)^2}{m(1 - m)} + \alpha + \alpha^2 - m\alpha \right) \\ &= -\frac{1}{2} \frac{\alpha}{m(1 - m)} (\alpha^2 + 2m\alpha(m - \alpha) + m^2) < 0.\end{aligned}$$

This implies that $\frac{d\alpha}{dm} = -\frac{\frac{\partial}{\partial m} g(\alpha, m)}{\frac{\partial}{\partial \alpha} g(\alpha, m)} < 0$. This proves (iii), which completes the proof. ■

Proof of Proposition 11. It is straightforward to show that in this case, the relevant results from Appendix B are still applicable. In particular, this means that for $\mu = 1$, there are only equilibria where both issues are chosen with a positive probability, and the line separating the two issues satisfies $\Phi_\Delta(x, y) = 0$, where

$$\begin{aligned}\Phi_\Delta(x, y) &= \xi_j^E \left((1 + \Delta)x + (1 - \Delta)\frac{y}{2} \right) + (1 - \xi_j^E) \tilde{a}_j^E \\ &\quad - (1 - \Delta) \left(\xi_j^F \left((1 - \Delta)y + (1 + \Delta)\frac{x}{2} \right) - (1 - \xi_j^F) \tilde{a}_j^F \right).\end{aligned}$$

Furthermore, $\mu = 1$ implies that voters get the same signals in sequential and simultaneous games, which in turn implies that the two politicians will use identical strategies. To find the equilibrium strategies, consider, without loss of generality, the case $\Delta \geq 0$ (the case $\Delta < 0$ is symmetric; notice that the case $\Delta = 0$ corresponds to the main setup of the model).

Since $\mu = 1$ implies $\xi_j^E = \xi_j^F = 1$ for each politician, we have, for indifferent types of politicians,

$$(1 + \Delta)x + (1 - \Delta)\frac{y}{2} = (1 - \Delta)y + (1 + \Delta)\frac{x}{2},$$

which is equivalent to $y = \frac{1 + \Delta}{1 - \Delta}x$. Thus, there is a unique equilibrium with strategies as described in the proposition.

Let us now compute social welfare. Again, let us focus on the case where $\Delta \geq 0$. As before, it

is given by (12). We have

$$\begin{aligned}
W &= 1 + 2A \left(\int_0^{\frac{1-\Delta}{1+\Delta}} \int_0^{\frac{1+\Delta}{1-\Delta}x} \left((1+\Delta)x + (1-\Delta)\frac{1}{2}\frac{1+\Delta}{1-\Delta}x \right)^2 dy dx \right. \\
&\quad + \int_{\frac{1-\Delta}{1+\Delta}}^1 \int_0^1 \left((1+\Delta)x + (1-\Delta)\frac{1}{2} \right)^2 dy dx \\
&\quad \left. \int_0^1 \int_0^{\frac{1-\Delta}{1+\Delta}y} \left((1-\Delta)y + (1+\Delta)\frac{1}{2}\frac{1-\Delta}{1+\Delta}y \right)^2 dx dy - 1 \right) \\
&= 1 + \frac{A}{12} \frac{3\Delta + 9\Delta^2 + \Delta^3 + 3}{\Delta + 1} = 1 + A \frac{(1+\Delta)^2}{12} \left(2 + \left(\frac{1-\Delta}{1+\Delta} \right)^3 \right).
\end{aligned}$$

This function is increasing and convex in Δ for $\Delta > 0$; for $\Delta = 0$ (equal weight) it equals $1 + \frac{A}{4}$, whereas for $\Delta = 1$ (only E matters) it equals $1 + \frac{2A}{3}$.

If instead the politicians are forced to campaign on E (the more important issue), the voters' social welfare will equal

$$1 + 2A \left(\int_0^1 \left((1+\Delta)x + (1-\Delta)\frac{1}{2} \right)^2 dx - 1 \right) = 1 + A \frac{(1+\Delta)^2}{6}.$$

It is straightforward to check that the social welfare in this case is strictly lower for $\Delta < 1$, and is the same only for the extreme case $\Delta = 1$ (in this case, only E matters to voters, and both politicians will campaign on it with probability 1). Naturally, fixing the less important issue will result in an even lower welfare (more precisely, it will equal $1 + A \frac{(1-\Delta)^2}{6}$). This completes the proof. ■

Proof of Proposition 12. Consider a politician, say, Incumbent. Let δ be the share of Challengers who choose both issues ($d_c = EF$); then, by symmetry, the shares of Challengers choosing E only and F only are $\frac{1-\delta}{2}$ each. In terms of δ , we have $\xi = \xi_i^E = \xi_i^F = \frac{1+\delta}{2} + \mu \frac{1-\delta}{2}$.

Take Incumbent with $e_i > f_i$; such Incumbent compares choosing $d_i = E$ with $d_i = EF$ and, by monotonicity, for a given e_i , a higher f_i makes him weakly more likely to campaign on both issues. It is straightforward to consider different cases and show that the separating line must be linear and connect points (α, α) and $(1, \beta)$ for $\beta \leq \alpha$. Suppose a type (x, y) is indifferent. The posterior impression of voters if he chooses to talk about both issues is:

$$\xi x + (1-\xi) \frac{1+x}{2} + \xi y + (1-\xi) \frac{1+y}{2},$$

and if he chooses to talk about E only, it equals

$$\xi \left(x + \frac{y}{2} \right) + (1 - \xi) \tilde{a}_i^E,$$

where

$$\tilde{a}_i^E = \frac{\frac{\alpha}{2} \left(\frac{2\alpha+1}{3} \right) + \frac{\beta(1-\alpha)}{2} \left(\frac{2\alpha+\beta+2}{3} \right)}{\frac{1-\delta}{2}} = \frac{\alpha(2\alpha+1) + \beta(1-\alpha)(2\alpha+\beta+2)}{3(1-\delta)}.$$

The separating line must therefore satisfy the equation

$$\xi x + (1 - \xi) \frac{1+x}{2} + \xi y + (1 - \xi) \frac{1+y}{2} - \xi \left(x + \frac{y}{2} \right) - (1 - \xi) \frac{\alpha(2\alpha+1) + \beta(1-\alpha)(2\alpha+\beta+2)}{3(1-\delta)} = \frac{M}{A}.$$

From this, we conclude that the slope of the separating line is $-(1 - \xi)$, which implies $(\alpha - \beta) = (1 - \xi)(1 - \alpha)$, so $\beta = \alpha - (1 - \xi)(1 - \alpha)$. Furthermore, since the behavior of Incumbent and Challenger are symmetric, we have $\delta = (1 - \alpha)(1 - \beta) = (1 - \alpha)(1 - \alpha + (1 - \xi)(1 - \alpha)) = (1 - \alpha)^2(2 - \xi)$. On the other hand, $\xi = \frac{1+\delta}{2} + \mu \frac{1-\delta}{2}$, which implies $\delta = \frac{(1-\alpha)^2(3-\mu)}{2+(1-\alpha)^2(1-\mu)}$ and $\beta = 1 - \frac{(1-\alpha)(3-\mu)}{2+(1-\alpha)^2(1-\mu)} = \frac{\alpha(1+\alpha+\mu-\alpha\mu)}{2+(1-\alpha)^2(1-\mu)}$, so $\xi = \frac{3-4\alpha-\mu+2\alpha^2+4\alpha\mu-2\alpha^2\mu}{2+(1-\alpha)^2(1-\mu)}$. Plugging β , δ , and ξ , and substituting (x, y) for (α, α) , we get the equation

$$\frac{\alpha(3-\mu)}{6 \left(2 + (1-\alpha)^2(1-\mu) \right)^2} \left[(1-\alpha)^2(6\alpha-4\alpha^2-11)\mu + (1-\alpha)^3(1-2\alpha)\mu^2 \right] = \frac{M}{A}. \quad (\text{A9})$$

For any given μ and A , the left-hand side is an increasing function of α , because its derivative equals

$$\frac{3-\mu}{6 \left(2 + (1-\alpha)^2(1-\mu) \right)^3} \left[\begin{aligned} & 12(\alpha + \mu - \alpha\mu) + \\ & 36(1-\mu)(1-\alpha)^2 + (1-\mu)^2(1-\alpha)^2 \times \\ & \left((1-\alpha)^2(10 + \mu + 2(1-\mu)(1-\alpha)^2) + 6\alpha \right) \end{aligned} \right] > 0.$$

If $\alpha = 0$ (in which case $\beta = 0$), then $M = 0$, and if $\alpha = 1$ (in which case $\beta = 1$), then $M = \frac{3-\mu}{4}A$. This proves that for $M = 0$, all or almost all types campaign on both issues, and for $M > \frac{3-\mu}{4}A$, each politician campaigns on one issue only. For $M \in \left(0, \frac{3-\mu}{4}A \right)$, there is exactly one interior solution $\alpha \in (0, 1)$, which is increasing in M . Finally, notice that $\delta = (1 - \alpha)^2(2 - \xi)$ and $\xi = \frac{1+\delta}{2} + \mu \frac{1-\delta}{2}$ together imply

$$\delta - \frac{3-\mu-(1-\mu)\delta}{2}(1-\alpha)^2 = 0,$$

and the left-hand side is increasing in both α and δ . Consequently, an increase in M results in a higher α and thus a lower δ . This completes the proof. ■

Proof of Proposition 13. For $\mu = 1$, $\frac{3-\mu}{4}A = \frac{A}{2}$, so if $M > \frac{A}{2}$, no politician talks about both issues. For $M \in \left[0, \frac{A}{2} \right]$, plugging $\mu = 1$ into (A9) yields $\frac{\alpha}{2} = \frac{M}{A}$, so $\alpha = \beta = 2\frac{M}{A}$, $\xi = 1$, and $\delta = \left(1 - 2\frac{M}{A} \right)^2$. The result on politicians' strategies follows.

Let us now compute voters' welfare. For each politician, the relevant integral in (12) equals

$$2 \int_0^\alpha \int_0^x \left(x + \frac{x}{2}\right)^2 dy dx + 2 \int_\alpha^1 \int_0^\alpha \left(x + \frac{\alpha}{2}\right)^2 dy dx + 2 \int_\alpha^1 \int_\alpha^x (x+y)^2 dy dx = \frac{1}{24} (-4\alpha^3 + 3\alpha^4 + 28),$$

so total voters' welfare equals

$$1 + \frac{A}{12} (4 - 4\alpha^3 + 3\alpha^4) = 1 + \frac{A}{3} + \frac{A}{12} \alpha^3 (3\alpha - 4) = 1 + \frac{A}{3} + \frac{4M^3}{3A^2} \left(3\frac{M}{A} - 2\right).$$

This is decreasing in M (the derivative equals $-8\frac{M^2}{A^3} (A - 2M) < 0$), from $1 + \frac{A}{3}$ if $M = 0$ (this corresponds to the full information benchmark from Subsection 3.2) to $1 + \frac{A}{4}$ if $M = \frac{A}{2}$ (this coincides with the case of simultaneous game if only campaigns on one issue are possible and $\mu = 1$, as in Proposition 6). Thus, voters' welfare is decreasing in M .

The politicians' joint welfare equals 1 (the utility of holding office), less the amount of money spent during the campaign, which equals $(1 - \alpha)^2 M = \left(1 - 2\frac{M}{A}\right)^2 M$ for each politician. Thus, the total expected welfare of politicians is

$$1 - 2 \left(1 - 2\frac{M}{A}\right)^2 M.$$

Its derivative with respect to M is $-2(A - 6M) \frac{A - 2M}{A^2}$, so it is decreasing on $M \in (0, \frac{A}{6})$ and increasing on $M \in (\frac{A}{6}, \frac{A}{2})$; notice that for both $M = 0$ and $M = \frac{A}{2}$ it equals 1. Its minimum, on the other hand, is at $M = \frac{A}{6}$ and is equal to $1 - \frac{4}{27}A$; in this case, $\alpha = \frac{1}{3}$, and thus probability $\delta = \left(1 - \frac{1}{3}\right)^2 = \frac{4}{9}$.

Let us now sum the voters' and politicians' utilities with weights z and $1 - z$, respectively. The derivative with respect to M is then

$$-2 \left(1 - \frac{2M}{A}\right) \left((1 - z) - 3(1 - z) \frac{2M}{A} + z \left(\frac{2M}{A}\right)^2 \right).$$

This is nonpositive for all $M \in [0, \frac{A}{2}]$ if and only if $z \geq \frac{9}{13}$, so politicians' weight is not greater than $\frac{4}{13}$. This completes the proof. ■

Proof of Proposition 14. The validity of formula (16) was established in the text. Checking its limit properties and verifying that choosing the same issue is always equilibrium for second movers, is straightforward algebra and is omitted. ■

Appendix B — For Online Publication

This Appendix establishes several auxiliary results. In particular, Lemma B2 establishes monotonicity of equilibrium strategies. This Appendix is not intended for publication.

Lemma B1 *In any equilibrium, $\hat{a}_j^E(x)$ and $\bar{a}_j^E(x)$ are weakly increasing in x , and $\hat{a}_j^F(y)$ and $\bar{a}_j^F(y)$ are weakly increasing in y .*

Proof. Consider Incumbent, and suppose he chooses E , then the voters' posterior belief about his competence is $\bar{a}_i^E(e_i) = \xi_i^E \hat{a}_i^E(e_i) + (1 - \xi_i^E) \bar{a}_i^E$, and the politician's expected utility is an affine transformation of this, per Lemma 1. Thus, $\bar{a}_i^E(x)$ is nondecreasing in x . Moreover, if $\xi_i^E > 0$, then $\hat{a}_i^E(x)$ is nondecreasing in x . Similarly, $\bar{a}_i^F(y)$ is nondecreasing in y , and $\hat{a}_i^F(y)$ is also nondecreasing, provided that $\xi_i^F > 0$.

In case of Challenger, we need to consider his decisions in the nodes where Incumbent chose $d_i = E$ and where he chose $d_i = F$ separately. Other than that, however, the reasoning is absolutely identical, so $\bar{a}_c^E(x; d_i = E)$ and $\bar{a}_c^E(x; d_i = F)$ are nondecreasing in x , while $\bar{a}_c^F(y; d_i = E)$ and $\bar{a}_c^F(y; d_i = F)$ are nondecreasing in y ; the results for $\hat{a}_c^F(y; d_i = E)$ and $\hat{a}_c^F(y; d_i = F)$ follow similarly. This completes the proof. ■

The next result establishes monotonicity of equilibrium strategies. Let λ_1 and λ_2 denote the one-dimensional and two-dimensional Lebesgue measures, respectively.

Lemma B2 *Suppose that for politician $j \in \{i, c\}$, $\hat{a}_j^E(x)$ and $\hat{a}_j^F(y)$ are weakly increasing in x and y , respectively. Then the strategies of politician j are monotone: for any $x_l \leq x \leq x_h$ and any $y_l \leq y \leq y_h$:*

$$\begin{aligned} (x, y) \in E_j \text{ implies } (x_h, y) \in E_j \text{ and } (x, y_l) \in E_j; \\ (x, y) \in F_j \text{ implies } (x_l, y) \in F_j \text{ and } (x, y_h) \in F_j. \end{aligned} \tag{B1}$$

Proof. Suppose, to obtain a contradiction, that this is not the case. Without loss of generality, this means that for some $x_1 < x_2$ and some y_0 , $(x_1, y_0) \in E_j$ and $(x_2, y_0) \in F_j$. Let $\hat{a}_j^E(x_1) = m$; then $\hat{a}_j^E(x_2) = m$ as well (indeed, $\hat{a}_j^E(x)$ is nondecreasing, and if $\hat{a}_j^E(x_2) > m$, the politician would have to choose F at (x_2, y_0) , because the expected utility from choosing F is the same at (x_1, y_0) and (x_2, y_0)). Now let $X = \{x : \hat{a}_j^E(x) = m\}$, then weak monotonicity of $\hat{a}_j^E(x)$ implies that X is an interval, with or without endpoints (denote them by x_l and x_h), and $[x_1, x_2] \subset X$.

Let $n = \hat{a}_j^F(y_0)$ and let $Y = \{y : \hat{a}_j^F(y) = n\}$. There are two possibilities. First, suppose that Y is a singleton $\{y_0\}$; then for $y < y_0$ and $x \in X$, the politician must choose E (so such $(x, y) \in E_j$), and for $y > y_0$ and $x \in X$, the politician must choose F (so such $(x, y) \in F_j$). If $y_0 > 0$, then for any $x \in X$, the set $\{y : (x, y) \in E_j\}$ is either $[0, y_0)$ or $[0, y_0]$, and in either case $\hat{f}_j(x) = \frac{y_0}{2}$. This, however, implies that $\hat{a}_j^E(x) = x + \hat{f}_j(x)$ is a strictly increasing function of x on X , contradicting the definition of X .

The other case is $y_0 = 0$. In this case, for any $x \in X$ and any $y > 0$, $(x, y) \in F_j$. Thus, $(x_1, y) \in E_j$ if and only if $y = y_0 = 0$. In equilibrium, therefore, beliefs must satisfy $\hat{f}_j(x_1) = 0$, which implies that

$$\hat{a}_j^E(x_2) = x_2 + \hat{f}_j(x_2) > x_1 + 0 = x_1 + \hat{f}_j(x_1) = \hat{a}_j^E(x_1).$$

But we proved above that $\hat{a}_j^E(x_1) = \hat{a}_j^E(x_2) = m$, so we got to a contradiction, which completes the proof.

Now consider the second possibility, where Y is not a singleton. Then it is an interval (similarly to X), with or without endpoints y_l and y_h . We have the following: if $y \in Y$, then $x < x_l$ implies that $\hat{a}_j^E(x) < m$ and $(x, y) \in F_j$, while $x > x_h$ implies $\hat{a}_j^E(x) > m$ and $(x, y) \in E_j$; similarly, if $x \in X$, then $y < y_l$ implies $\hat{a}_j^F(y) < n$ and $(x, y) \in E_j$, while $y > y_h$ implies $\hat{a}_j^F(y) > n$ and $(x, y) \in F_j$; finally, $x \in X$ and $y \in Y$ implies $\hat{a}_j^E(x) = m$ and $\hat{a}_j^F(y) = n$.

Consider three possible cases. First, suppose that $x_h - x_l > y_h - y_l$ (i.e., X has a larger linear measure than Y). Take $x'_l, x'_h \in X$ such that $x'_h - x'_l > y_h - y_l$ (x_l and x_h might not be in X). If $y_l = 0$; then the only values of y where $(x'_l, y) \in E_j$ or $(x'_h, y) \in E_j$ are possible are $y \in [y_l, y_h]$. Thus, equilibrium beliefs satisfy $\hat{f}_j(x'_l) \leq y_h$ and $\hat{f}_j(x'_h) \geq y_l = 0$, but this implies that

$$\hat{a}_j^E(x'_l) \leq x'_l + y_h < x'_h + y_l = \hat{a}_j^E(x'_h),$$

a contradiction. If, on the other hand, $y_l > 0$, consider the following: if ν is the linear measure of $(\{x'_l\} \times Y) \cap E_j$, then the expected value of that set is at most $y_h - \frac{\nu}{2}$, and $\hat{f}_j(x'_l) \leq \frac{y_l}{y_l + \nu} \frac{y_l}{2} + \frac{\nu}{y_l + \nu} (y_h - \frac{\nu}{2}) < y_h - \frac{y_l}{2}$ for all $\nu \in [0, y_h - y_l]$ (indeed, the difference is $\frac{2y_l(y_h - y_l) + \nu(\nu - y_l)}{2(y_l + \nu)}$, and the numerator is a convex function of ν , so it suffices to check the sign at $\nu \in \{0, y_h - y_l, \frac{y_l}{2}\}$, the latter only if $\frac{y_l}{2} < y_h - y_l$, which is equivalent to $y_h > \frac{3}{2}y_l$. We get $2y_l(y_h - y_l) > 0$ for $\nu = 0$, $y_h(y_h - y_l) > 0$ for $\nu = y_h - y_l$, and $\frac{1}{4}y_l(8y_h - 9y_l) > \frac{1}{4}y_l(8 \times \frac{3}{2}y_l - 9y_l) = \frac{3}{4}y_l > 0$ for $\nu = \frac{y_l}{2}$). At the same time, $\hat{f}_j(x'_h) \geq \frac{y_l}{2}$, and thus

$$\hat{a}_j^E(x'_l) \leq x'_l + y_h - \frac{y_l}{2} < x'_h + y_l - \frac{y_l}{2} = x'_h + \frac{y_l}{2} \leq \hat{a}_j^E(x'_h),$$

again a contradiction. This proves that $x_h - x_l > y_h - y_l$ is impossible.

Second, suppose that $x_h - x_l < y_h - y_l$. A similar reasoning will lead to a similar contradiction due to symmetry of sets X and Y .

Third, suppose that $x_h - x_l = y_h - y_l$. Careful examination of the argument above shows that we would get a contradiction even in this case (by choosing x'_l and x'_h sufficiently close to x_l and x_h and exploiting strict inequalities), unless $x_l = y_l = 0$. Take a small $\varepsilon < \frac{y_h}{3}$ and consider the set $[0, \varepsilon] \times [0, \varepsilon]$. Without loss of generality, suppose that with this square, the relative measure of points where the politician chooses E is at least half (i.e., $\lambda_2([0, \varepsilon] \times [0, \varepsilon] \cap E_j) \geq \frac{\varepsilon^2}{2}$). For this to be true, there must be some $x'_l \in (0, \varepsilon)$ such that $\lambda_1(\{x'_l\} \times [0, \varepsilon] \cap E_j) > \frac{\varepsilon}{2}$. If so, $\hat{f}_j(x'_l)$

must satisfy $\hat{f}_j(x'_l) < y_h - \varepsilon$ (to show this, suppose that $\nu_l = \lambda_1(\{\{x'_l\} \times [0, \varepsilon]\} \cap E_j) \geq \frac{\varepsilon}{2}$ and $\nu_h = \lambda_1(\{\{x'_l\} \times [\varepsilon, y_h]\} \cap E_j) \leq y_h$, then $\hat{f}_j(x'_l) \leq \frac{\nu_l}{\nu_l + \nu_h} \varepsilon + \frac{\nu_h}{\nu_l + \nu_h} (y_h - \frac{\nu_h}{2})$; now, the difference $y_h - \varepsilon - \left(\frac{\nu_l}{\nu_l + \nu_h} \varepsilon + \frac{\nu_h}{\nu_l + \nu_h} (y_h - \frac{\nu_h}{2}) \right) = \frac{\nu_h(\nu_h - 2\varepsilon) + 2\nu_l(y_h - 2\varepsilon)}{2(\nu_h + \nu_l)} \geq \frac{-\varepsilon^2 + 2\frac{\varepsilon}{2}(y_h - 2\varepsilon)}{2(\nu_h + \nu_l)} = \frac{\varepsilon(y_h - 3\varepsilon)}{2(\nu_h + \nu_l)} > 0$, so $\hat{f}_j(x'_l) < y_h - \varepsilon$). At the same time, $\hat{f}_j(x_h) \geq 0$, therefore,

$$\hat{a}_j^E(x'_l) \leq x'_l + y_h - \varepsilon < \varepsilon + y_l - \varepsilon = y_l + 0 \leq \hat{a}_j^E(x_h),$$

a contradiction. We have thus considered all possibilities; this completes the proof. ■

Lemmas B1 and B2 together imply that any equilibrium satisfies monotonicity in strategies, in the sense of (B1).

Lemma B3 *For each politician j , G_j is a simple curve which may be parametrized by a (closed) interval $T \subset [0, 2]$ such that $t \in T$ is mapped into (x, y) such that $x + y = t$.*

Proof. Suppose first that both E_j and F_j have positive area measures. Then there is a point $(x, y) \in E_j$ with $x < 1$ and $y > 0$, and thus point $(1, 0)$ and all points (x, y) within some ε -radius of $(1, 0)$ belong to E_j ; similarly, all points within ε -radius of $(0, 1)$ belong to F_j . Since G is the intersecion of two closed sets, it is closed. Define $t_1 = \min_{(x,y) \in G} (x + y)$ and $t_2 = \max_{(x,y) \in G} (x + y)$ and let $\phi : G \rightarrow [t_1, t_2]$ be defined by $\phi(x, y) = x + y$. It is injective: indeed, if for two points $(x_1, y_1), (x_2, y_2) \in G$, we had $x_1 < x_2$ and $y_1 > y_2$, then there would be points $(x_3, y_3) \in E_j$ and $(x_4, y_4) \in F_j$ with $x_3 < x_4$ and $y_3 > y_4$ (because we could take these points arbitrarily close to (x_1, y_1) and (x_2, y_2) , respectively). But this would contradict Lemma B2: indeed, monotonicity would imply that point $(x_4, y_3) \in E_j$ because $x_4 > x_3$, but also that $(x_4, y_3) \in F_j$ because $y_3 > y_4$, which is impossible. At the same time, it is also surjective. Indeed, take some $t \in (t_1, t_2)$ which is not part of $\text{Im } \phi$. Since $\text{Im } \phi$ is a closed set (as ϕ is a continuous function with a compact as its range), take $t_3 = \max[\text{Im } \phi \cap [t_1, t]]$ and $t_4 = \min[\text{Im } \phi \cap [t, t_2]]$. Let $(x_5, y_5) = \phi^{-1}(t_3)$ and $(x_6, y_6) = \phi^{-1}(t_4)$ and let $\varepsilon = \min(x_6 - x_5, y_6 - y_5) > 0$. Within $\frac{\varepsilon}{2}$ -neighborhood of (x_5, y_5) , pick $(x_7, y_7) \in E_j$ and within $\frac{\varepsilon}{2}$ -neighborhood of (x_6, y_6) , pick $(x_8, y_8) \in F_j$. Then, by monotonicity, $(x_8, y_7) \in E_j$, and $(x_7, y_8) \in F_j$ (if not, then $(x_8, y_8) \in E_j$ by monotonicity, a contradiction). Now, observe that

$$\begin{aligned} x_8 + y_7 &> x_6 - \frac{\varepsilon}{2} + y_5 - \frac{\varepsilon}{2} = x_6 - \varepsilon + y_5 \geq x_5 + y_5 = t_1, \\ x_8 + y_7 &< x_6 + \frac{\varepsilon}{2} + y_5 + \frac{\varepsilon}{2} = x_6 + (y_5 + \varepsilon) < x_6 + y_6 = t_2, \end{aligned}$$

and thus $\phi(x_8, y_7) \in (t_1, t_2)$; similarly, $\phi(x_7, y_8) \in (t_1, t_2)$. Thus, for any $\gamma \in [0, 1]$, we have $\phi(\gamma x_8 + (1 - \gamma)x_7, \gamma y_7 + (1 - \gamma)y_8) \in (t_1, t_2)$. However, there is γ for which this point is in $\bar{E}_j \cap \bar{F}_j$, a contradiction.

The remaining case is where either E_j or F_j , say E_j , has measure zero. If $E_j \neq \emptyset$, then $(x, y) \in E_j$ implies that either $y = 0$ or $x = 1$. Monotonicity of strategies then trivially implies that

G is a connected subset of $[0, 1] \times \{0\} \cup \{1\} \times [0, 1]$, and the result immediately follows. If $E_j = \emptyset$, then G is empty, and the statement is trivially true. The same argument works if F_j has measure zero. This completes the proof. ■

The following lemma characterizes the cases where the projections of G on the two axes are one-to-one mappings.

Lemma B4 *Let $(x, y) \in G \setminus \partial\Omega_j$ and suppose that $(e_j, y) \in G \Rightarrow e_j = x$ and $(x, f_j) \in G \Rightarrow f_j = y$, i.e., there are no other points in G with the same e_j or f_j . Then $\Phi(x, y) = 0$.*

Proof. Suppose, to obtain a contradiction, that $\Phi(x, y) \neq 0$. Suppose that $\Phi(x, y) > 0$ (the opposite case is analogous). Take $\varepsilon = \frac{\Phi(x, y)}{3}$; then, since curve G is continuous, there exists $\delta \in (0, \varepsilon)$ for which there exists a unique point $(x', y') \in G$ with $x' = x - \delta$ and $y' \in (y - \varepsilon, y)$ (let $\varepsilon' = y - y' < \varepsilon$), and also $\Phi(x', y) > \Phi(x, y) - \varepsilon$. Now, consider a politician with type (x', y) ; we know, by definition of G , that $(x', y) \in F$, and, by continuity of Φ , that $\Phi(x', y) > 0$. By construction, we have $\hat{e}_j(y) = \frac{x}{2}$, and $\hat{f}_j(x) = \frac{y'}{2}$. Consequently, if this politician chooses E , the voters' posterior is $\bar{a}_j^E(x') = \xi_j^E \left(x' + \frac{y'}{2}\right) + (1 - \xi_j^E) \tilde{a}_j^E$, and if he chooses F , the posterior is $\bar{a}_j^F(y) = \xi_j^F \left(y + \frac{x}{2}\right) + (1 - \xi_j^F) \tilde{a}_j^F$. But $(x', y) \in F$, thus $\bar{a}_j^F(y) \geq \bar{a}_j^E(x')$. Note, however, that

$$\begin{aligned} \bar{a}_j^E(x') - \bar{a}_j^F(y) &= \Phi(x, y) + \xi_j^E \left(x' - x + \frac{y' - y}{2}\right) \\ &= \Phi(x, y) - \xi_j^E \left(\delta + \frac{\varepsilon'}{2}\right) \\ &> \Phi(x, y) - \varepsilon - \left(\varepsilon + \frac{\varepsilon}{2}\right) \\ &> \Phi(x, y) - 3\varepsilon = 0, \end{aligned}$$

We get a contradiction which completes the proof. ■

The next lemma considers the case where (x, y) lies on a horizontal or vertical segment of G .

Lemma B5 *Suppose that G contains a vertical segment not entirely on the border: for some $\alpha \in (0, 1)$, $\{y : (\alpha, y) \in G\} = [\beta, \gamma]$. Then $\Phi(\alpha, \beta) \leq 0$ and $\Phi(\alpha, \gamma) \geq 0$, in particular, for some $y \in [\beta, \gamma]$, $\Phi(\alpha, y) = 0$. Similarly, suppose that G contains a horizontal segment not entirely on the border: for some $\alpha \in (0, 1)$, $\{x : (x, \alpha) \in G\} = [\beta, \gamma]$. Then $\Phi(\beta, \alpha) \geq 0$ and $\Phi(\gamma, \alpha) \leq 0$, in particular, for some $x \in [\beta, \gamma]$, $\Phi(x, \alpha) = 0$.*

Proof. Suppose that G has a vertical segment with (x, y) where $x = \alpha$, $y \in [\beta, \gamma]$ and suppose $\alpha \in (0, 1)$. If there is $\tilde{y} \in (\beta, \gamma)$ such that $(\alpha, y) \in E$ for $y < \tilde{y}$ and $(\alpha, y) \in F$ for $y > \tilde{y}$, then it must be that $\Phi(\alpha, \tilde{y}) = 0$. Indeed, a politician (α, y) with $y \in (\beta, \gamma)$ expects to get (in terms of voters' posterior belief) $\bar{a}_j^E(\alpha) = \xi_j^E \left(\alpha + \frac{\tilde{y}}{2}\right) + (1 - \xi_j^E) \tilde{a}_j^E$ from choosing E and to get

$\bar{a}_j^F(y) = \xi_j^F(y + \frac{\alpha}{2}) + (1 - \xi_j^F)\tilde{a}_j^F$ from choosing F . Now, we have $\bar{a}_j^E(\alpha) \geq \bar{a}_j^F(y)$ for y close to \tilde{y} but less than \tilde{y} and $\bar{a}_j^E(\alpha) \leq \bar{a}_j^F(y)$ for y close to \tilde{y} but greater than \tilde{y} . Taking limits, we get $\Phi(\alpha, \tilde{y}) = 0$.

Consider the case $\beta > 0$. Take a small ε and consider the politician $(x', y') = (\alpha - \varepsilon, \beta + \varepsilon)$. If ε is small enough, then there is some β' such that $(\alpha - \varepsilon, \beta') \in G$; for almost all ε , β' is unique (take such ε) and, moreover, $0 < \beta' < \beta$. If we take ε small enough, then β' will be arbitrarily close to β . Now, $\hat{f}_j(x') = \frac{\beta'}{2}$ and $\hat{e}_j(y') = \frac{\alpha}{2}$. This politician chooses F in equilibrium. At the same time, he expects to get $\bar{a}_j^E(\alpha - \varepsilon) = \xi_j^E(\alpha - \varepsilon + \frac{\beta'}{2}) + (1 - \xi_j^E)\tilde{a}_j^E$ if he chooses E and to get $\bar{a}_j^F(\beta + \varepsilon) = \xi_j^F(\beta + \varepsilon + \frac{\alpha}{2}) + (1 - \xi_j^F)\tilde{a}_j^F$. Taking the limit $\varepsilon \rightarrow 0$, we get $\Phi(\alpha, \beta) \leq 0$.

Now consider the case $\beta = 0$. In this case, again take a small ε and consider the politician $(x', y') = (\alpha - \varepsilon, \varepsilon)$. Then $\hat{f}_j(x') \geq 0$ and $\hat{e}_j(y') = \frac{\alpha}{2}$. This player chooses F in equilibrium. At the same time, he expects to get $\bar{a}_j^E(\alpha - \varepsilon) = \xi_j^E(\alpha - \varepsilon + \frac{\hat{f}_j(x')}{2}) + (1 - \xi_j^E)\tilde{a}_j^E$ if he chooses E and to get $\bar{a}_j^F(\varepsilon) = \xi_j^F(\varepsilon + \frac{\alpha}{2}) + (1 - \xi_j^F)\tilde{a}_j^F$. Consequently, we have $\xi_j^E(\alpha - \varepsilon) + (1 - \xi_j^E)\tilde{a}_j^E \leq \xi_j^F(\varepsilon + \frac{\alpha}{2}) + (1 - \xi_j^F)\tilde{a}_j^F$; taking the limit $\varepsilon \rightarrow 0$, we again get $\Phi(\alpha, \beta) \leq 0$.

We can similarly prove that $\Phi(\alpha, \gamma) \geq 0$. Thus, in any case, $\Phi(\alpha, \beta) \leq 0$ and $\Phi(\alpha, \gamma) \geq 0$ and therefore there is $(\alpha, y) \in G$ with $\Phi(\alpha, y) = 0$.

In the case of a horizontal segment $y = \alpha$, $x \in [\beta, \gamma]$, we can similarly prove that $\Phi(\beta, \alpha) \geq 0$ and $\Phi(\gamma, \alpha) \leq 0$, and thus there is (x, α) with $\Phi(x, \alpha) = 0$. ■

In what follows, let $r_j = \xi_j^E/\xi_j^F \in [\mu, \frac{1}{\mu}]$. The next results shows that for $\frac{1}{2} < r < 2$, G is precisely the set of points with $\Phi(x, y) = 0$.

Lemma B6 *Suppose that both E_j and F_j have positive measures. If $r \in (\frac{1}{2}, 2)$, then $\{(x, y) \in \Omega_j \setminus \partial\Omega_j : \Phi(x, y) = 0\} = G \setminus \partial\Omega_j$, and it is a upward-sloping line with slope $\frac{2r-1}{2-r}$. If $r \leq \frac{1}{2}$, then $G \setminus \partial\Omega_j$ is a horizontal line, and if $r \geq 2$, then $G \setminus \partial\Omega_j$ is a vertical line.*

Proof. We have $G \setminus \partial\Omega_j \neq \emptyset$. Suppose $r \in (\frac{1}{2}, 2)$. Then the set of points with $\Phi(x, y) = 0$ is an upward-sloping straight line with slope $\frac{2r-1}{2-r}$. Moreover, $\Phi(x, y)$ is strictly increasing in x and strictly decreasing in y . Consequently, G may not contain horizontal or vertical segments, as this would contradict Lemma B5. If so, Lemma B4 implies that all points $(x, y) \in G \setminus \partial\Omega_j$ satisfy $\Phi(x, y) = 0$.

If $r = 2$, then the set of points with $\Phi(x, y) = 0$ defines a vertical line. In this case, G cannot have a non-vertical upward-sloping part (by Lemma B4), and every vertical segment must lie on the set $\Phi(x, y) = 0$. A horizontal segment could have one end on the line $\Phi(x, y) = 0$. If this is the left end, then Lemma B5 implies that $\Phi(x, y) \leq 0$ on the right end. However, this is impossible, since Φ is strictly increasing in x in this case. We would get a similar contradiction if the right end of the horizontal segment satisfied $\Phi(x, y) = 0$. Thus, G is a vertical line.

If $r > 2$, then the set of points with $\Phi(x, y) = 0$ defines a downward sloping line. This means that G may have only one point of intersection with this set, and then Lemma B4 implies that G must be either a horizontal or a vertical line. If it is horizontal, then its left end should satisfy $\Phi(x, y) \geq 0$ and its right end should satisfy $\Phi(x, y) \leq 0$ by Lemma B5. Again, this contradicts the fact that Φ is strictly increasing in x . Thus, G is a vertical line in this case, too.

The cases $r = \frac{1}{2}$ and $r < \frac{1}{2}$ are similar. ■

This result suggests that $G \setminus \partial\Omega_j$ is always a line (perhaps empty): upward-sloping, vertical, or horizontal. Lemma B10 below extends this result by showing that the same is true for G (inclusive of $\partial\Omega_j$), unless almost all politicians choose E_j or almost all choose F_j . Before that, however, we prove some necessary and sufficient conditions for existence of different types of equilibria.

The next Lemma characterizes the conditions under which the curve G separating regions E and F may be a horizontal or a vertical line.

Lemma B7 *There exists an equilibrium for which the line $G \setminus \partial\Omega_j$ separating regions E and F is the vertical line connecting points $(\alpha, 0)$ and $(\alpha, 1)$, where $0 < \alpha < 1$ if and only if $\alpha \in \left[1 + \frac{\xi_j^F}{\xi_j^E} - \frac{1}{\xi_j^E}, 2 - \frac{\xi_j^F}{\xi_j^E} - \frac{1}{\xi_j^E}\right]$, which is only possible if $r \geq 2$. Similarly, there exists an equilibrium for which the line $G \setminus \partial\Omega_j$ separating regions E and F is the horizontal line connecting points $(0, \beta)$ and $(1, \beta)$, where $0 < \beta < 1$ if and only if $\beta \in \left[1 + \frac{\xi_j^E}{\xi_j^F} - \frac{1}{\xi_j^F}, 2 - \frac{\xi_j^E}{\xi_j^F} - \frac{1}{\xi_j^F}\right]$, which is only possible if $r \leq \frac{1}{2}$.*

Proof. It suffices to prove the first part of the result, as the proof of the second part is completely symmetric.

Under the conditions of the Lemma, we have $\tilde{a}_j^E = \frac{\alpha+1}{2} + \frac{1}{2} = \frac{\alpha+2}{2}$ and $\tilde{a}_j^F = \frac{\alpha}{2} + \frac{1}{2} = \frac{\alpha+1}{2}$. For (x, y) such that $x > \alpha$, we have $\hat{f}_j(x) = \frac{1}{2}$; for $x < \alpha$, $\hat{f}_j(x)$ may take different values. At the same time, for any (x, y) , $\hat{e}_j(y) = \frac{\alpha}{2}$. For any (x, y) with $x < \alpha$, and $y > 0$, we must have

$$\xi_j^E \left(x + \hat{f}_j(x)\right) + (1 - \xi_j^E) \frac{\alpha + 2}{2} \leq \xi_j^F \left(y + \frac{\alpha}{2}\right) + (1 - \xi_j^F) \frac{\alpha + 1}{2}; \quad (\text{B2})$$

in particular, this should hold for x arbitrarily close to α and y arbitrarily close to 0. Since $\hat{f}_j(x) \geq 0$, we have

$$\xi_j^E \alpha + (1 - \xi_j^E) \frac{\alpha + 2}{2} \leq \xi_j^F \frac{\alpha}{2} + (1 - \xi_j^F) \frac{\alpha + 1}{2}. \quad (\text{B3})$$

For any (x, y) with $x > \alpha$, we must have

$$\xi_j^E \left(x + \frac{1}{2}\right) + (1 - \xi_j^E) \frac{\alpha + 2}{2} \geq \xi_j^F \left(y + \frac{\alpha}{2}\right) + (1 - \xi_j^F) \frac{\alpha + 1}{2}; \quad (\text{B4})$$

in particular, this should be true for y arbitrarily close to 1 (and x arbitrarily close to α). Therefore,

$$\xi_j^E \left(\alpha + \frac{1}{2}\right) + (1 - \xi_j^E) \frac{\alpha + 2}{2} \geq \xi_j^F \left(1 + \frac{\alpha}{2}\right) + (1 - \xi_j^F) \frac{\alpha + 1}{2}. \quad (\text{B5})$$

Now, (B3) is equivalent to $\xi_j^E \alpha \leq 2\xi_j^E - \xi_j^F - 1$, and (B5) is equivalent to $\xi_j^E \alpha \geq \xi_j^E + \xi_j^F - 1$. Since $\xi_j^E > 0$, this implies $\alpha \in \left[1 + \frac{\xi_j^F}{\xi_j^E} - \frac{1}{\xi_j^E}, 2 - \frac{\xi_j^F}{\xi_j^E} - \frac{1}{\xi_j^E}\right]$. One can easily verify that for any such α (provided that $\alpha \in (0, 1)$) there is an equilibrium, where we define $\hat{f}_j(x) = 0$ for $x < \alpha$, whereas for $x = \alpha$, we let $(x, y) \in E$ if and only if $y < \gamma$, where γ satisfies

$$\xi_j^E \left(\alpha + \frac{\gamma}{2}\right) + (1 - \xi_j^E) \frac{\alpha + 2}{2} = \xi_j^F \left(\gamma + \frac{\alpha}{2}\right) + (1 - \xi_j^F) \frac{\alpha + 1}{2} \quad (\text{B6})$$

(existence of such value follows from inequalities (B3) and (B5); it is unique if $\xi_j^E > 2\xi_j^F$). Finally, the interval for α is nonempty if and only if $2\frac{\xi_j^F}{\xi_j^E} < 1$, which is equivalent to $r \geq 2$. ■

The next Lemma characterizes conditions under which there is an equilibrium where (almost) all types choose E .

Lemma B8 *There exists an equilibrium where almost all types choose E if and only if $\frac{1}{2}\xi_j^E + \xi_j^F \leq 1$, and there exists an equilibrium where almost all types choose F if and only if $\xi_j^E + \frac{1}{2}\xi_j^F \leq 1$.*

Proof. It suffices to consider the case where all types of politician, except perhaps for a set of measure zero, choose E . We have $\tilde{a}_j^E = \frac{1}{2} + \frac{1}{2} = 1$. For any (x, y) , we have $\hat{f}_j(x) = \frac{1}{2}$, whereas $\hat{e}_j(y)$ may take different values, as may \tilde{a}_j^F . Politician (x, y) can choose E in equilibrium if and only if

$$\xi_j^E \left(x + \frac{1}{2}\right) + (1 - \xi_j^E) \geq \xi_j^F (y + \hat{e}_j(y)) + (1 - \xi_j^F) \tilde{a}_j^F.$$

This must be satisfied for (x, y) arbitrarily close to $(0, 1)$; we can find suitable $\hat{e}_j(y)$ and \tilde{a}_j^F (for example, zeros) if and only if

$$\xi_j^E \left(\frac{1}{2}\right) + (1 - \xi_j^E) \geq \xi_j^F.$$

Thus, a necessary condition for such equilibrium is $1 - \frac{1}{2}\xi_j^E - \xi_j^F \geq 0$. At the same time, since the type $(0, 1)$ is most prone to deviation, this condition is also sufficient for the existence of an equilibrium where all types choose E to exist. ■

The next Lemma restricts the possible separation lines G if $r \in (\frac{1}{2}, 2)$.

Lemma B9 *Suppose $\frac{1}{2} < r < 2$, and $\max(\xi_j^E, \xi_j^F) \geq \frac{2}{3}$. Furthermore, suppose that E_j and F_j have positive measures. Then there exists $(x, y) \in G \setminus \partial\Omega_j$ such that $x = y$.*

Proof. Consider the case $1 \leq r < 2$ (the case $\frac{1}{2} < r \leq 1$ is considered similarly). In this case, $\max(\xi_j^E, \xi_j^F) = \xi_j^E$. By Lemma B4, all points $(x, y) \in G \setminus \partial\Omega_j$ satisfy $\Phi(x, y) = 0$. Solving the equation $\Phi(x, y) = 0$ for y , we get

$$y = \frac{2r - 1}{2 - r}x + 2 \frac{(1 - \xi_j^E) \tilde{a}_j^E - (1 - \xi_j^F) \tilde{a}_j^F}{2\xi_j^F - \xi_j^E}. \quad (\text{B7})$$

For $1 \leq r < 2$, this defines an upward-sloping line with slope $\frac{2r-1}{2-r} \in (0, 1]$.

Suppose that all $(x, y) \in G \setminus \partial\Omega_j$ satisfy $x > y$. Then we can denote the points of intersection of G with $\partial\Omega_j$ by $(\alpha, 0)$ and $(1, \beta)$ (with $\alpha < 1$, $\beta > 0$ because of the assumption of positive measure; also, $(\alpha, \beta) = (0, 1)$ is also ruled out, as then G would be a line with all points satisfying $x = y$). If so, we have that $\beta = \frac{2r-1}{2-r}(1 - \alpha)$. Then we have

$$\tilde{a}_j^E = \frac{2}{3} + \frac{\alpha}{3} + \frac{\beta}{3} = \frac{2}{3} + \frac{1 - \beta \frac{2-r}{2r-1}}{3} + \frac{\beta}{3} = 1 + \frac{r-1}{2r-1}\beta.$$

Since $r \geq 1$, $\tilde{a}_j^E \geq 1$. On the other hand, in this case, $\Pr(d_j = E) = \frac{\beta(1-\alpha)}{2} \leq \frac{1}{2} \leq \Pr(d_j = F)$, and thus $\tilde{a}_j^F \leq 1$. We can rewrite $\Phi(x, y) = 0$ as

$$\xi_j^E \left(x + \frac{y}{2} - \tilde{a}_j^E \right) - \xi_j^F \left(y + \frac{x}{2} - \tilde{a}_j^F \right) + \tilde{a}_j^E - \tilde{a}_j^F = 0, \quad (\text{B8})$$

which must be true for any (x, y) for which the indifference condition is satisfied and, in particular, for points on G arbitrarily close to $(\alpha, 0)$. Let us show that the left-hand side is actually positive. A sufficient condition for that is

$$\xi_j^E (\alpha - \tilde{a}_j^E) - \xi_j^F \left(\frac{\alpha}{2} - 1 \right) + \tilde{a}_j^E - 1 > 0 \quad (\text{B9})$$

Substituting $\alpha = 1 - \frac{2-r}{2r-1}\beta$ and $\tilde{a}_j^E = 1 + \frac{r-1}{2r-1}\beta$, the left-hand side equals

$$\xi_j^E \left(-\frac{\beta}{2r-1} \right) + \frac{1}{2}\xi_j^F \left(1 + \frac{2-r}{2r-1}\beta \right) + \frac{r-1}{2r-1}\beta.$$

This is positive if $\beta = 0$, while if $\beta > 0$, this equals

$$\xi_j^F \left(-r + \frac{1}{2} \left(\frac{2r-1}{\beta} + 2 - r \right) \right) + r - 1 > 0.$$

This last expression is positive if $r > 1$ (then the left-hand side is at least $\frac{1}{2}\xi_j^F(1-r) + r - 1$, which is then positive) and for $r = 1$, in which case it equals $\frac{1}{2}\xi_j^F \frac{1-\beta}{\beta}$, which is also positive, unless $\beta = 1$. However, $r = 1$ and $\beta = 1$ would imply $\alpha = 0$, and this combination is ruled out. Thus, we get a contradiction which shows that it is impossible that all $(x, y) \in G \setminus \partial\Omega_j$ satisfy $x > y$.

Now, suppose that all $(x, y) \in G \setminus \partial\Omega_j$ satisfy $x < y$. Then we can denote the points of intersection of G with $\partial\Omega_j$ by $(\alpha, 1)$ and $(0, \beta)$ (with $\alpha > 0$, $\beta < 1$ because of the assumption of positive measure). If so, we have $1 - \beta = \frac{2r-1}{2-r}\alpha$. We also have

$$\tilde{a}_j^F = \frac{\alpha}{3} + \frac{2}{3} + \frac{\beta}{3} = \frac{\alpha}{3} + \frac{2}{3} + \frac{1 - \frac{2r-1}{2-r}\alpha}{3} = 1 - \frac{r-1}{2-r}\alpha.$$

Since $1 \leq r < 2$, it must be that $\tilde{a}_j^F \leq 1$. As before, since $\Pr(d_j = F) = \frac{\alpha(1-\beta)}{2} \leq \frac{1}{2} \leq \Pr(d_j = E)$, we must have $\tilde{a}_j^E \geq 1$ and $1 - \tilde{a}_j^F \geq \tilde{a}_j^E - 1$, so $\tilde{a}_j^E \leq 2 - \tilde{a}_j^F$. In equilibrium, we must have (B8) satisfied for all (x, y) for which the indifference condition holds and, in particular, for points

arbitrarily close to $(0, \beta)$. Let us show that the left-hand side is actually negative. A sufficient condition for that is

$$\xi_j^E \left(\frac{\beta}{2} - 2 + \tilde{a}_j^F \right) - \xi_j^F (\beta - \tilde{a}_j^F) + 2(1 - \tilde{a}_j^F) < 0$$

Substituting $\beta = 1 - \frac{2r-1}{2-r}\alpha$ and $\tilde{a}_j^F = 1 - \frac{r-1}{2-r}\alpha$, the left-hand side equals

$$\frac{1}{2}\xi_j^E \left(-\frac{4r-3}{2-r}\alpha - 1 \right) + \xi_j^F \left(\frac{r}{2-r}\alpha \right) + 2\frac{r-1}{2-r}\alpha.$$

This is negative whenever

$$\frac{1}{2}\xi_j^F r \left(-(4r-3) - \frac{2-r}{\alpha} + 2 \right) + 2(r-1) < 0. \quad (\text{B10})$$

Now consider two subcases. If $r > 1$, then, since $\alpha \leq \frac{2-r}{2r-1}$ (because $\beta = 1 - \frac{2r-1}{2-r}\alpha \geq 0$), (B8) holds if

$$-3r(r-1)\xi_j^F + 2(r-1) < 0, \quad (\text{B11})$$

which is negative, provided that $\xi_j^F > \frac{2}{3r}$ and $r > 1$. On the other hand, if $r = 1$, then $\beta = 1 - \alpha$, and $x < y$ for all points on G implies $\alpha < \frac{2-r}{2r-1}$ strictly. Hence (B8) holds if (B11) is satisfied weakly, which is true since $r = 1$ in this case. Therefore, since we assumed that $\max(\xi_j^E, \xi_j^F) = \xi_j^E = \xi_j^F r > \frac{2}{3}$, we get to a contradiction. This completes the proof. ■

We finally prove that G is a straight line, provided that both issues are chosen with positive probability.

Lemma B10 *Suppose that E_j and F_j have positive measures. If $r \geq 2$, then G is a vertical straight line, if $r \leq \frac{1}{2}$, it is a horizontal straight line, and if $r \in (\frac{1}{2}, 2)$, it is an upward-sloping straight line with slope $\frac{2r-1}{2-r}$.*

Proof. Since both E_j and F_j have positive measures, $G \setminus \partial\Omega_j$ must be nonempty. Suppose first that $r \geq 2$. By Lemma B6, this implies that $G \setminus \partial\Omega_j$ is a vertical line connecting points $(\alpha, 0)$ and $(\alpha, 1)$ for some $\alpha \in (0, 1)$. To show that G contains no other points (other than $G \setminus \partial\Omega_j$ and the two endpoints), suppose, to obtain a contradiction, that a segment connecting $(\gamma, 0)$ and $(\alpha, 0)$ lies on G , for some γ satisfying $0 \leq \gamma < \alpha$. This implies that $(x, 0) \in E_j$ for all $x > \gamma$. This means that for $x \in (\alpha, \gamma)$,

$$\xi_j^E \left(x + \hat{f}_j(x) \right) + (1 - \xi_j^E) \frac{\alpha + 2}{2} \geq \xi_j^F (0 + \hat{e}_j(0)) + (1 - \xi_j^F) \frac{\alpha + 1}{2}.$$

But $\hat{f}_j(x) = 0$ for such x (because politicians (x, y) with $y > 0$ choose F). Moreover, since this inequality must hold for x arbitrarily close to γ , and $\hat{e}_j(0) \geq \frac{\gamma}{2}$ (this holds as equality if $\gamma > 0$ and is trivially true if $\gamma = 0$), it must be that

$$\xi_j^E \gamma + (1 - \xi_j^E) \frac{\alpha + 2}{2} \geq \xi_j^F \frac{\gamma}{2} + (1 - \xi_j^F) \frac{\alpha + 1}{2}.$$

Since such equilibrium is only possible if $r \geq 2$ (by Lemma B7), then, in particular, $\xi_j^E > \frac{1}{2}\xi_j^F$, and we can substitute γ for α and get a strict inequality

$$\xi_j^E \alpha + (1 - \xi_j^E) \frac{\alpha + 2}{2} > \xi_j^F \frac{\alpha}{2} + (1 - \xi_j^F) \frac{\alpha + 1}{2}.$$

Notice, however, that this contradicts (B3), which is a necessary condition for equilibrium.

We have thus proved that a segment connecting $(\gamma, 0)$ and $(\alpha, 0)$ cannot lie on G . We can similarly prove that a segment connecting $(\alpha, 1)$ and $(\delta, 1)$ for $\alpha < \delta \leq 1$ cannot lie on G either. This proves the result for $r \geq 2$. The proof for the case $r \leq \frac{1}{2}$ is similar.

Now, suppose that $r \in (\frac{1}{2}, 2)$. Then Lemma B6 implies that $G \setminus \partial\Omega_j$ is an upward-sloping line with slope $\frac{2r-1}{2-r}$. The proof that $G \cap \partial\Omega_j$ contains only the endpoints of this line is similar. This completes the proof. ■